Cours 2024-2025:

La perception des graphiques: un nouvel exemple de recyclage neuronal

The perception of graphics : a new example of neuronal recycling

Stanislas Dehaene
Chaire de Psychologie Cognitive Expérimentale

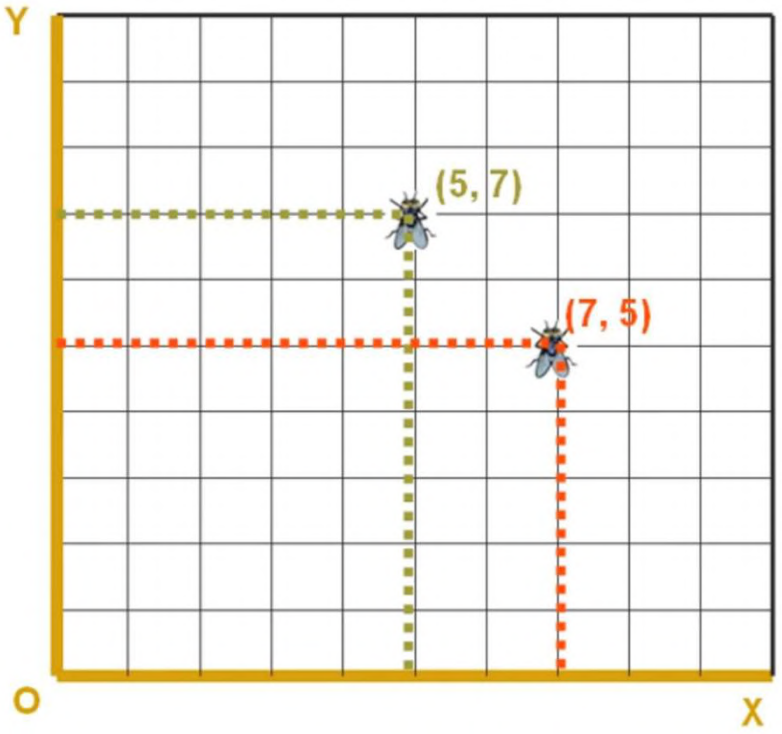
Cours n°3

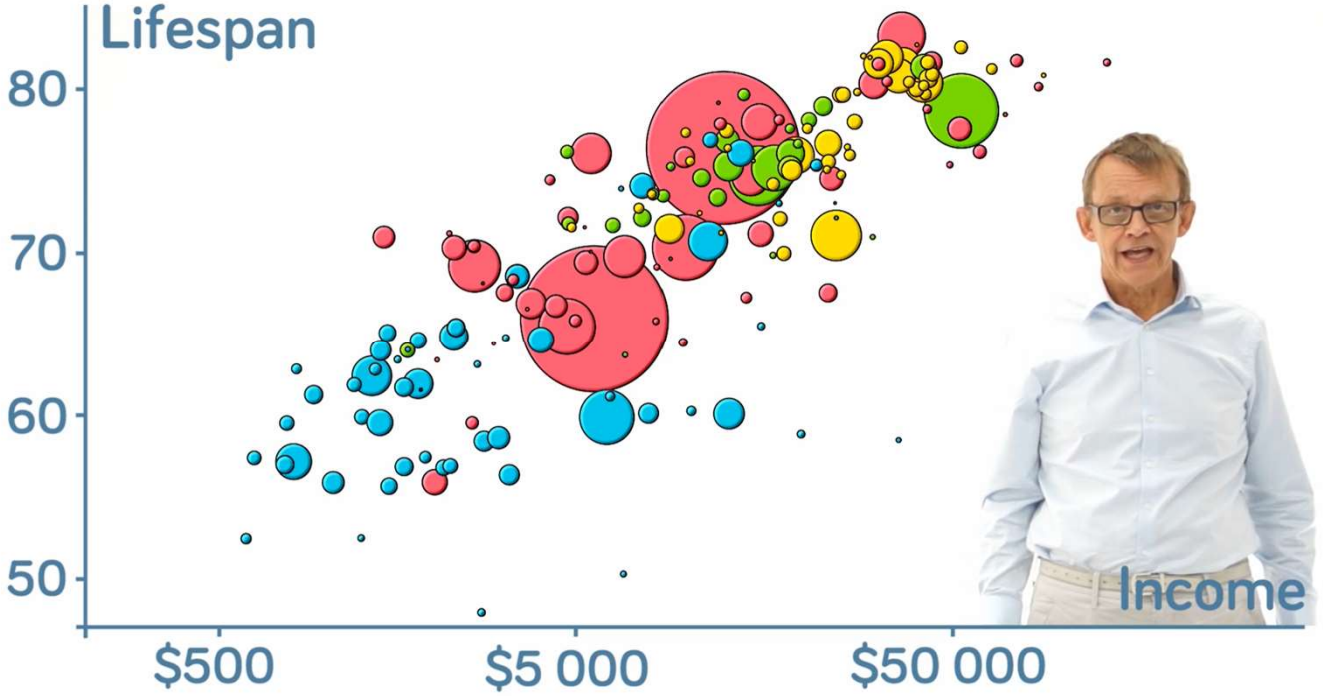
La correspondance entre le nombre et l'espace

The mapping between numbers and space

Cartesian coordinates:
Understanding that numbers can be used to map space

According to Encyclopedia Britannica: “According to legend, Descartes was inspired to devise his coordinate system by a fly he saw crawling on a tiled ceiling. He was curious how he might use numbers to describe the position of the fly and decided one way would be to choose a corner of the ceiling and count the tiles both horizontally and vertically from that corner to the fly. The result would be two numbers expressing exactly which tile the fly was on.”





What is a graphic?

Pinker, S. (1990). A Theory of Graph Comprehension. In *Artificial Intelligence and the Future of Testing*. Psychology Press.

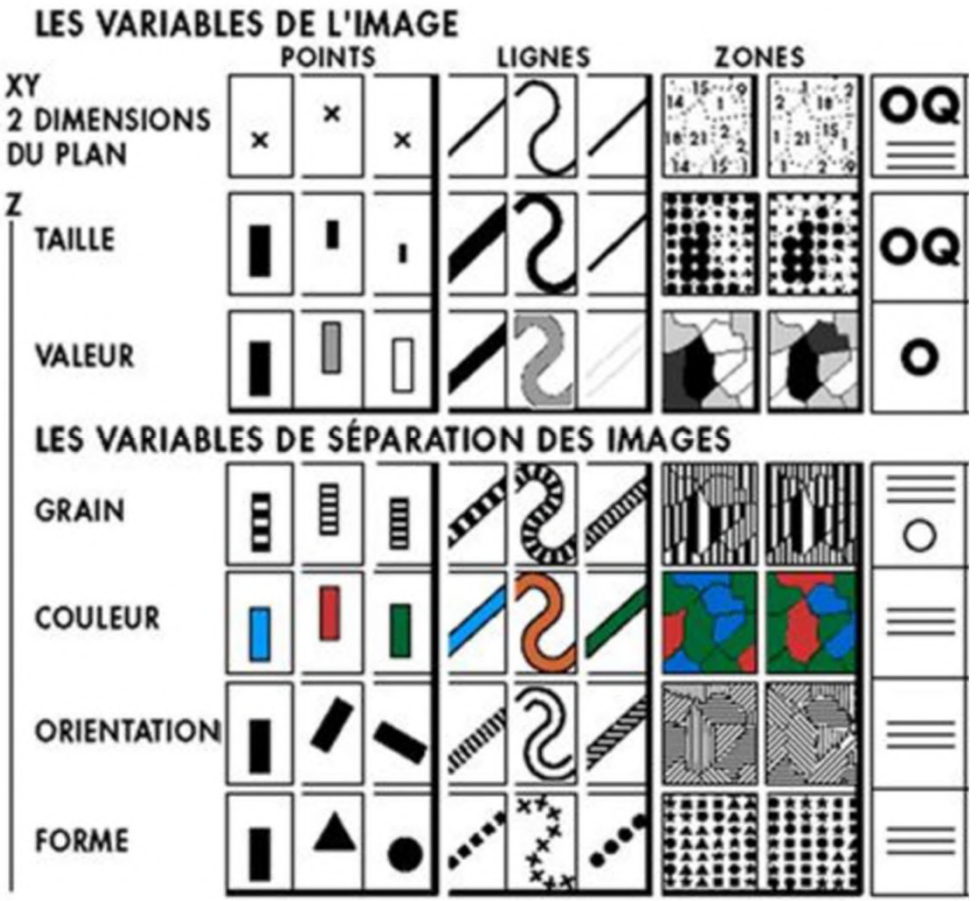
Cf Bertin (1967); or Pinker (1990):

« Each graph tries to communicate to the reader

- a set of n-tuples of values on n mathematical **scales**,
- using objects whose **visual dimensions** (i.e., length, position, lightness, shape, etc.) correspond to the respective scales
- and whose values on each dimension (i.e., an object's particular length, position, and so on) correlate with the values on the **corresponding scales**."

Graphics can be distinguished by their use of different types of scales

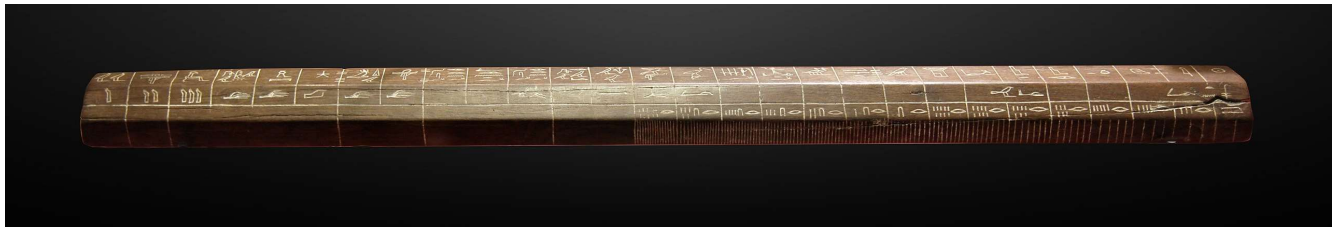
- Categorical (e.g., France, Germany…)
- Ordinal (e.g. Olympic results)
- Additive scale (e.g. temperature; numbers that can added or subtracted, but not divided)
- Ratio scale (e.g. weight)

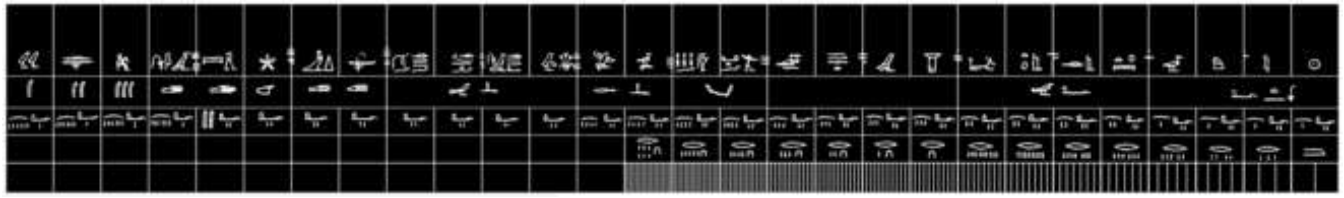


The spatial representation of quantities: an old idea

The idea that number can be used to measure space probably emerged multiple times:

- Measuring distances by counting one's steps (which even ants do!)
- Sticks graduated with a unit, e.g. the cubit (*coudée)*
- The first numerical displays: the nilometer, the clepsydra





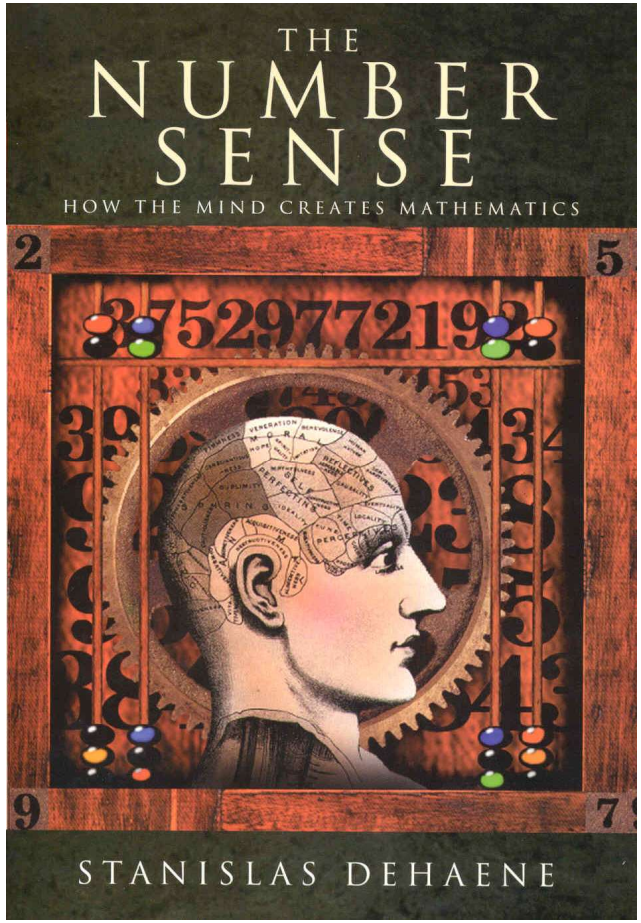
Clepsydra

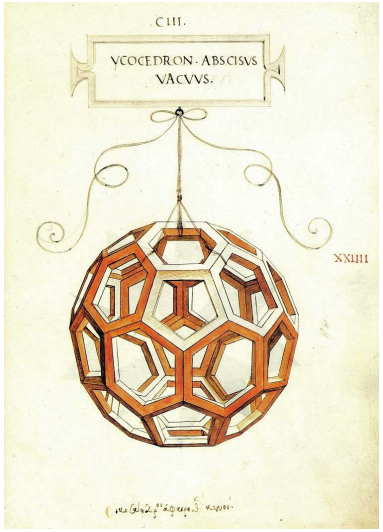
The nilometer in Alexandria

Space, Time and Number:

three fundamentally intertwined foundations of mathematics

The representation of numbers as a « line » play an essential role in the construction of mathematics.





The serendipitous discovery of the SNARC effect

(*Spatial-Numerical Association of Response Codes)*

Stanislas Dehaene, Serge Bossini and Pascal Giraux 1993

A study initially designed to investigate the semantics of numbers:

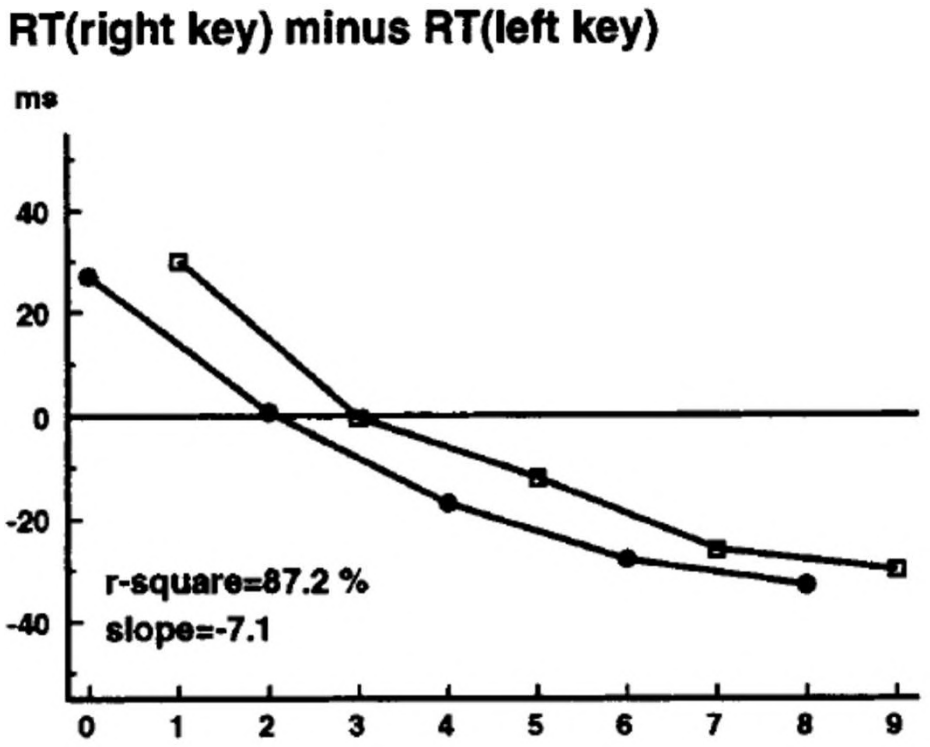
How do we know that 4 is even and 5 is odd?

Task = click a button for even numbers, another for odd digits.

Balanced assignment of the « odd » and « even » responses to the left and right hands in different blocks.

A triple interaction of block, parity, and magnitude…

The SNARC effect : an association of small numbers with the left and large numbers with the right.

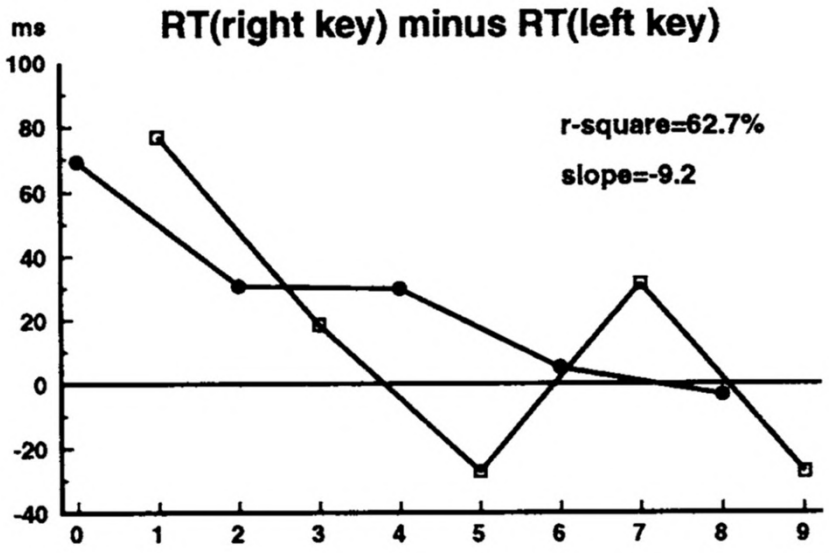


Faster with the left hand

Faster with the right hand

The high reproducibility of the SNARC effect

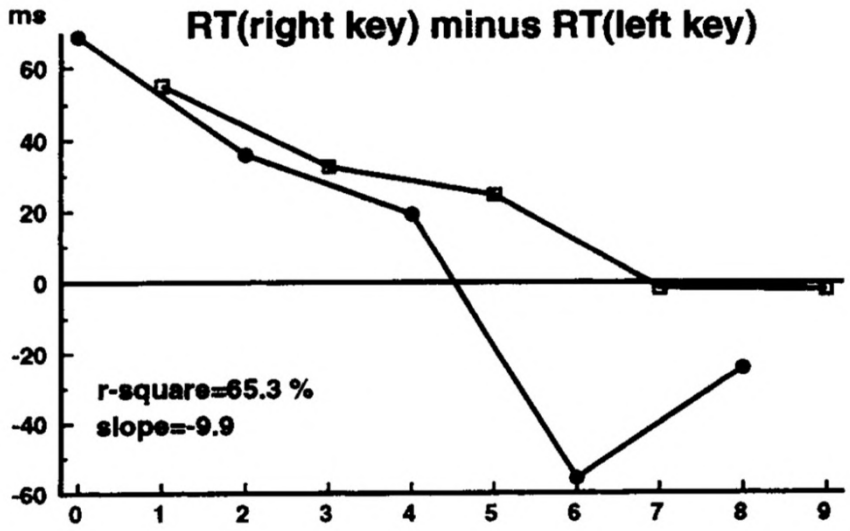
The effect is identical in left-handers.



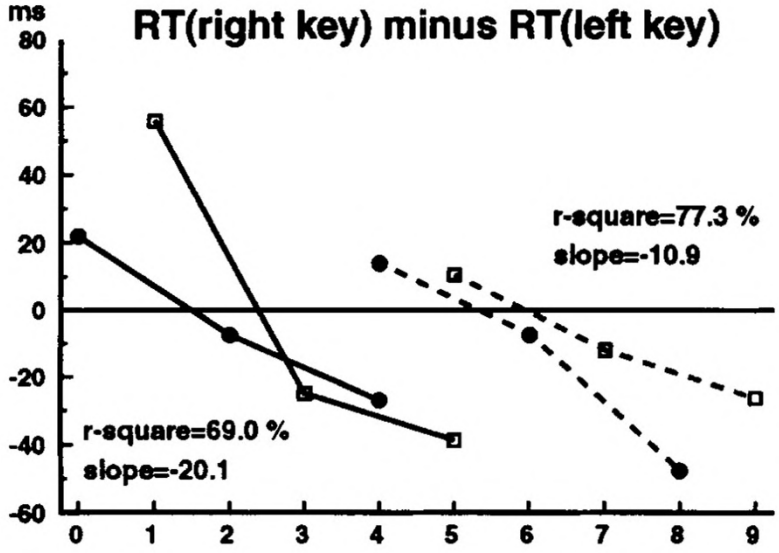
« what I tell you three times is true »

the Bellman

It depends on the side of space, not the hand (crossed-hands)



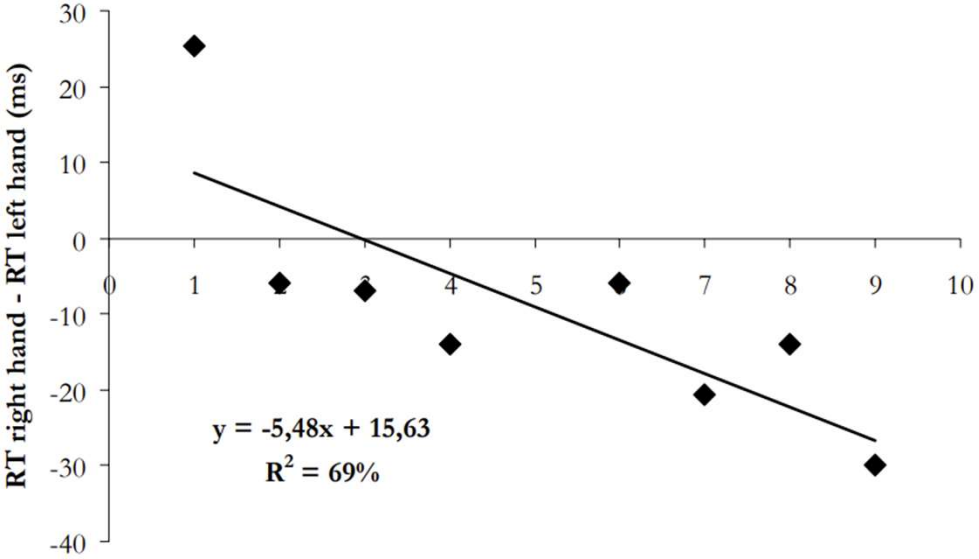
It depends on the ***relative*** size of numbers, not their absolute value.



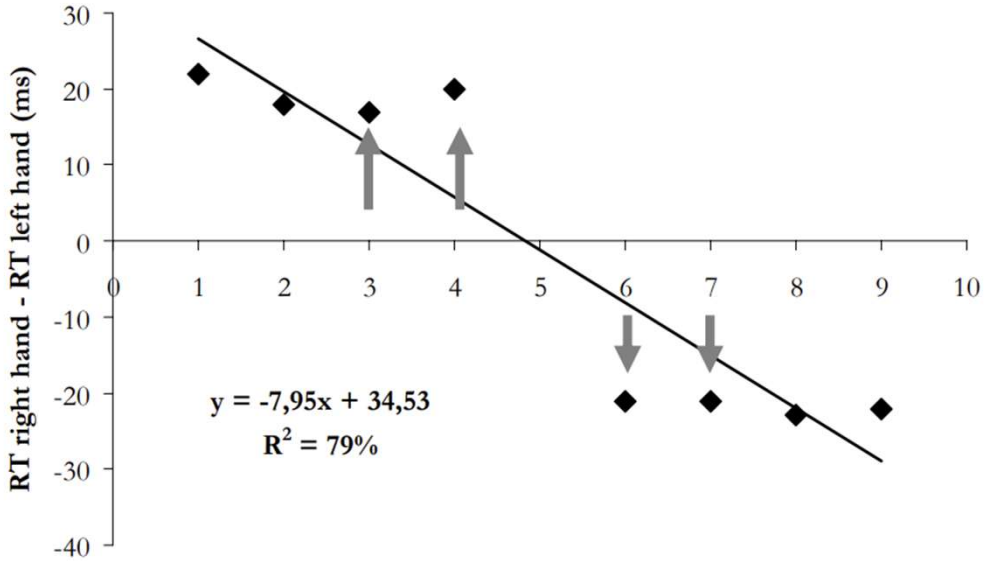
Slightly different SNARC effects in parity judgment and number comparison

Guida, A., & Campitelli, G. (2019). Explaining the SPoARC and SNARC effects with knowledge structures : An expertise account. *Psychonomic Bulletin & Review*, *26*(2), 434-451. https://doi.org/10.3758/s13423-019-01582-0

parity judgment



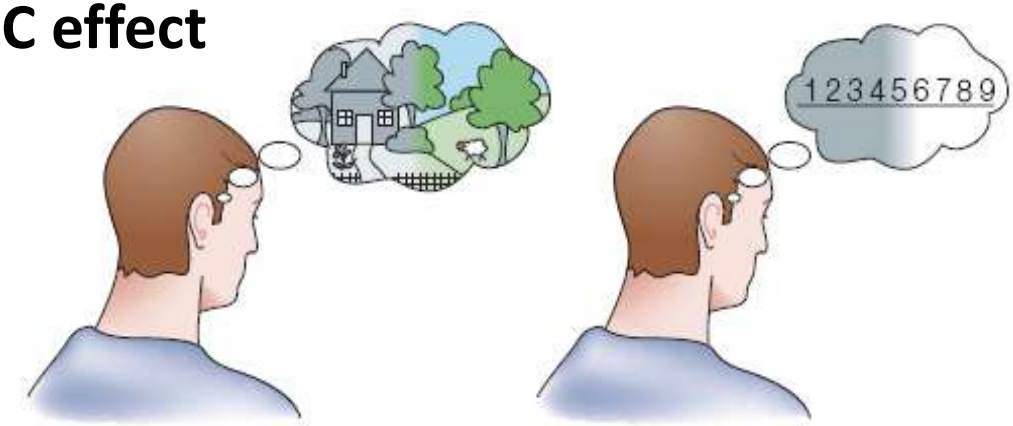
number comparison with 5



Some remarkable extensions of the SNARC effect

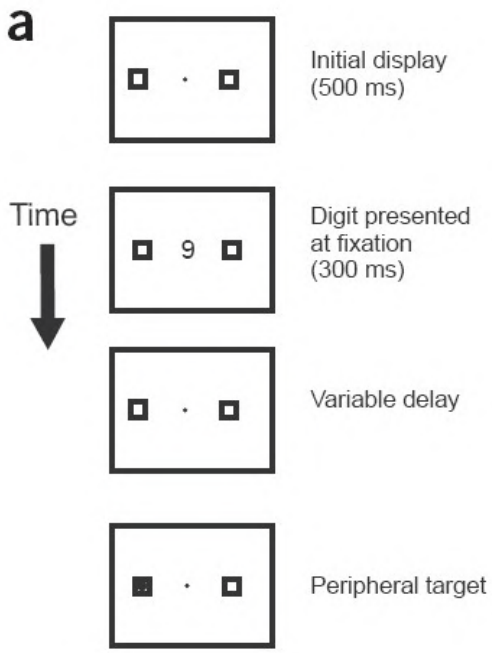
Zorzi, Priftis, & Umiltà, *Nature,* 2002:

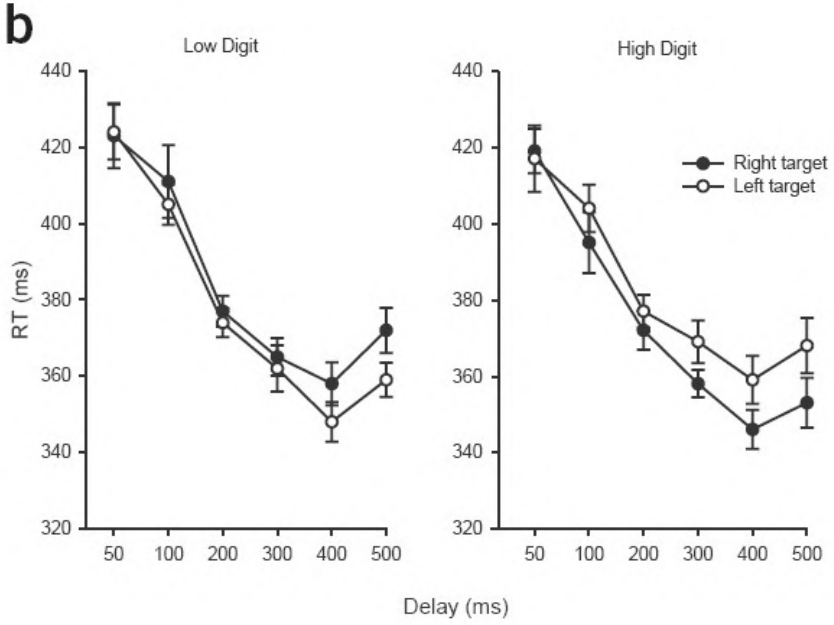
Hemineglect patients fail in bisecting two numbers: they may place 17 in the middle of 11 and 19!



Fischer et al., *Nature Neuroscience*, 2003:

The detection of a flash is faster in the right hemifield after a large central digit, and in the left hemifield after a small central digit





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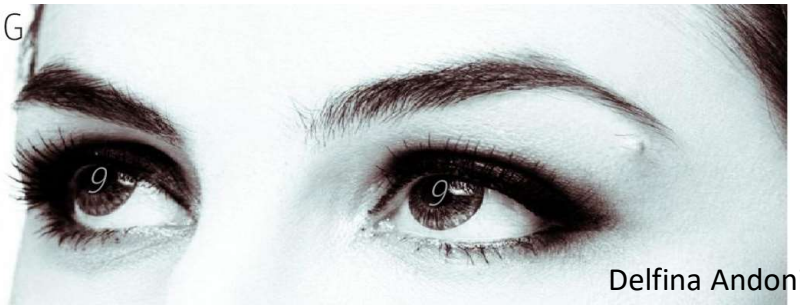
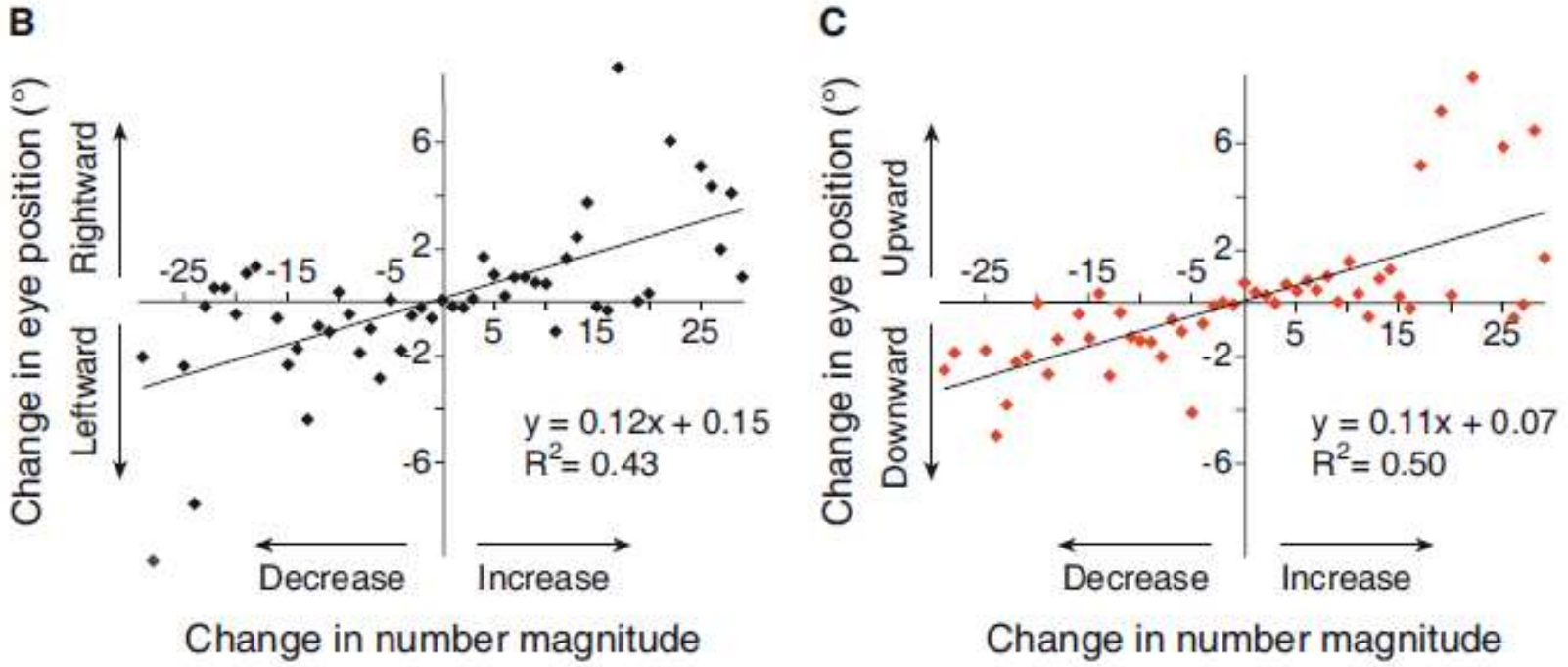


Delfina Andoni

Loetscher et al., *Current Biology*, 2010:

During the generation of random numbers in darkness, human adults move their eyes rightwards and upwards for larger numbers.



Ordinal and cardinal influences on the SNARC effect

van Dijck, J.-P., & Fias, W. (2011). A working memory account for spatial–numerical associations. *Cognition*, *119*(1), 114-119.

Before the parity judgment task, participants are asked to memorize a list of 5 random digits, in order.

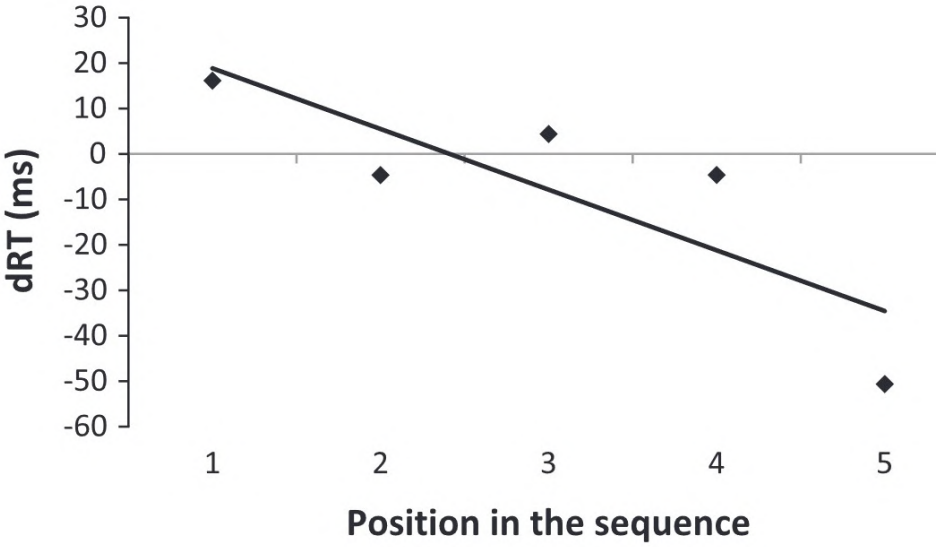
To ensure that they retain the list in working memory, they are then presented with the ten digits 0-9, but only perform the parity judgment task on the 5 memorized digits.

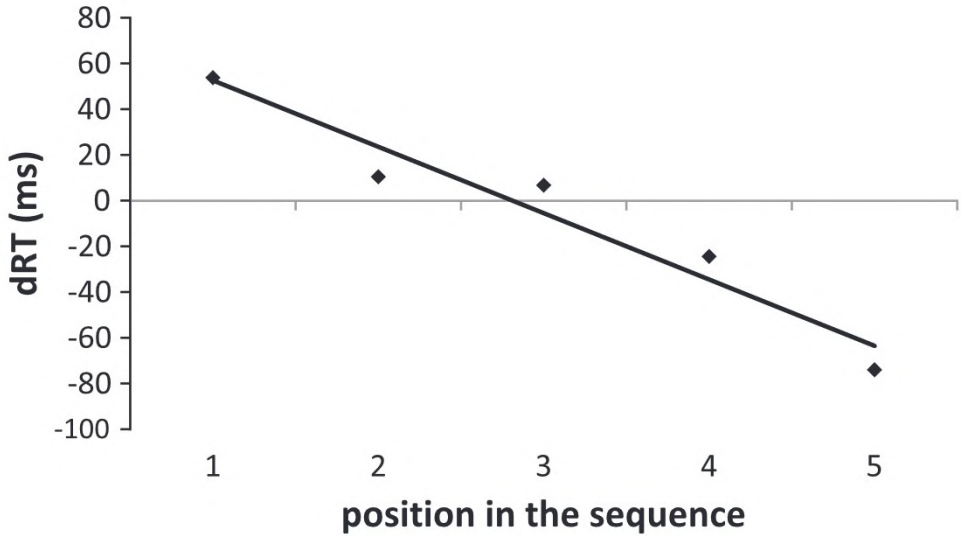
At the end of the block, they choose the correct sequence among 4 .

Result: no SNARC effect as a function of number magnitude, but solely as a function of ordinal position.

Idem with a non-numerical list (5 fruits and vegetables).

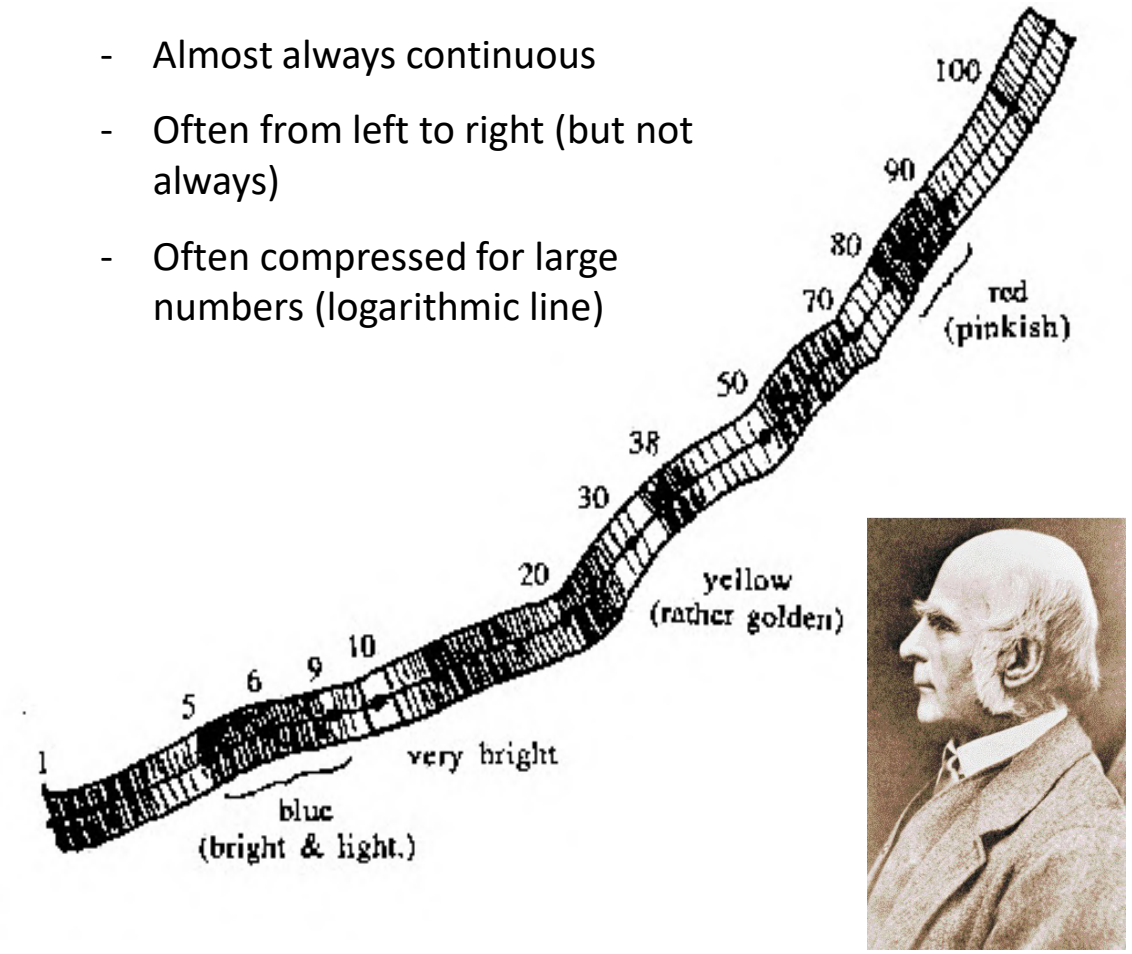
Conclusion: both ordinal and cardinal number can influence the SNARC – and one can override the other.





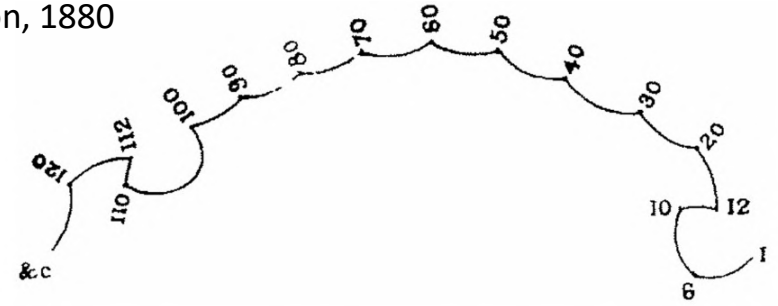
Conscious « number forms » : another link between number and space

- A conscious representation
- Almost always continuous
- Often from left to right (but not always)
- Often compressed for large numbers (logarithmic line)

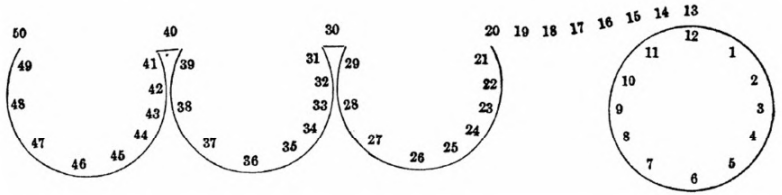


Sir Francis Galton, 1880, *Nature*

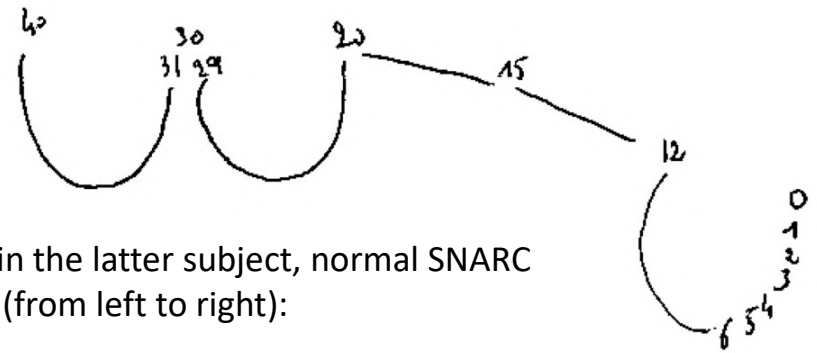
Galton, 1880



Calkins, 1892



Piazza et al., 2006



But... in the latter subject, normal SNARC effect (from left to right):

Dissociation between implicit and explicit representations of numbers?

Is the SNARC effect of cultural origin?

Does reading direction matter?

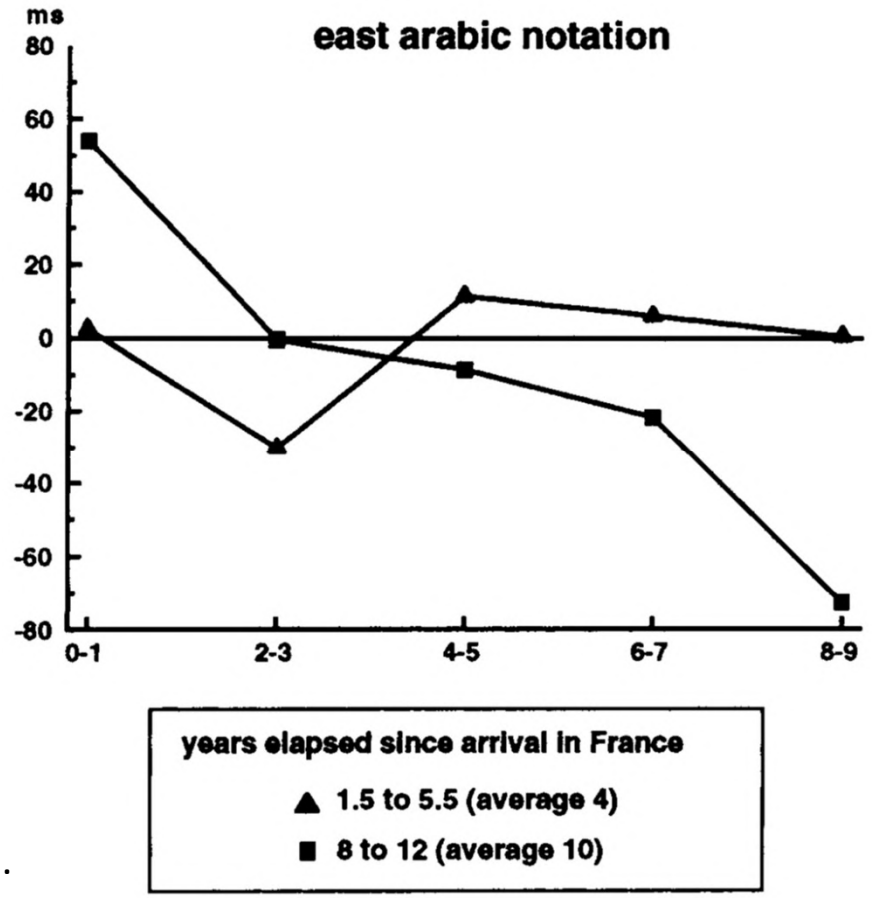
In our princeps paper, we tested Iranian subjects, who write from right to left, and found that effect tended to vanish for those who had arrived in France recently.

We supposed that reading direction (right-to-left) had an influence, but that it was partially cancelled by graphic conventions in math (left-to-right).

The conclusion was further supported by other studies: in **right-to-left readers**, the SNARC effect tends to disappear (Dehaene et al., 1993) or even reverse (Zebian, 2005; Hung et al., 2008; Shaki et al, 2008, 2009, 2012).

Other studies show that the effect is flexible within subjects, and that reading direction is just one of the biases that determines it – for instance, imagining a clock can reverse the SNARC effect.

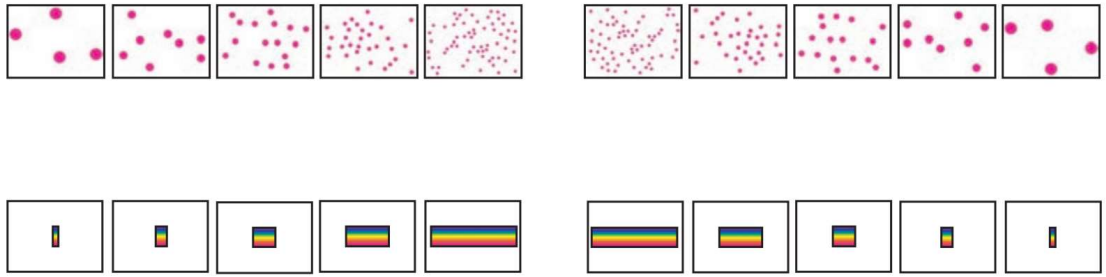
In children, with parity judgment, the SNARC effect appears around 3rd grade (Berch et al., 1999), around the time that children automatize the links between numbers and space (Girelli, JECP 2000).



A correspondence between number and space in infants !

de Hevia, M. D., Izard, V., Coubart, A., Spelke, E. S., & Streri, A. (2014). Representations of space, time, and number in neonates. *PNAS*, *111*(13), 4809-4813. https://doi.org/10.1073/pnas.1323628111

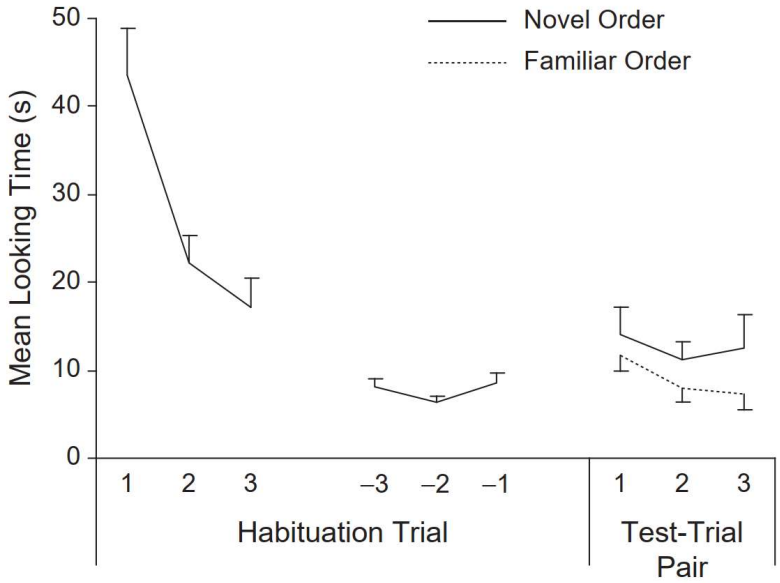
de Hevia, M. D., & Spelke, E. S. (2010). Number-space mapping in human infants. *Psychol Sci*, *21*(5), 653-660. https://doi.org/10.1177/0956797610366091

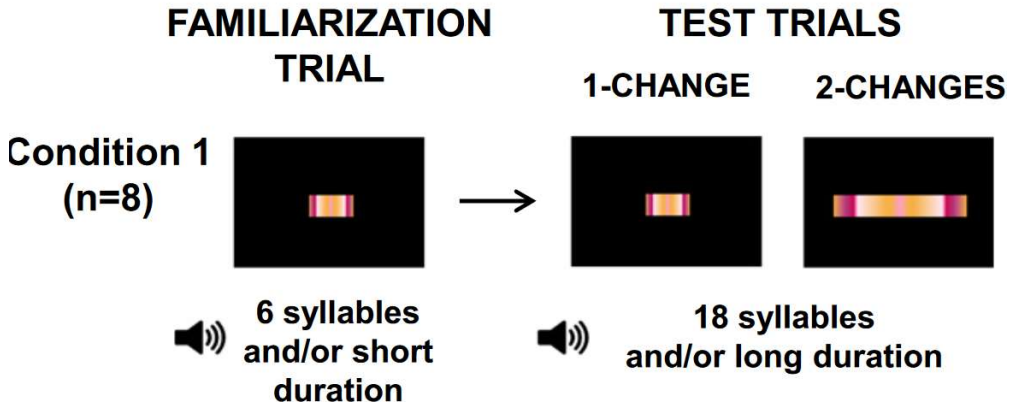


Infants and even 0-3 day-old neonates expect a systematic relationship between numerical and spatial magnitudes : large numbers correspond to long lines.

For instance, when 8-month-olds see a sequence of increasing numerosities, they dishabituated to a novel order where line lengths decreases rather than increases (and vice versa) (de Hevia et al., 2010).

Even when habituated to a single pairing of a midsize bar with an auditory numerosity, neonates preferentially look at the longer bar when the number is increased, and at the shorter bar when it is increased (de Hevia et al. 2014) .





A left-to-right number-space association in infants

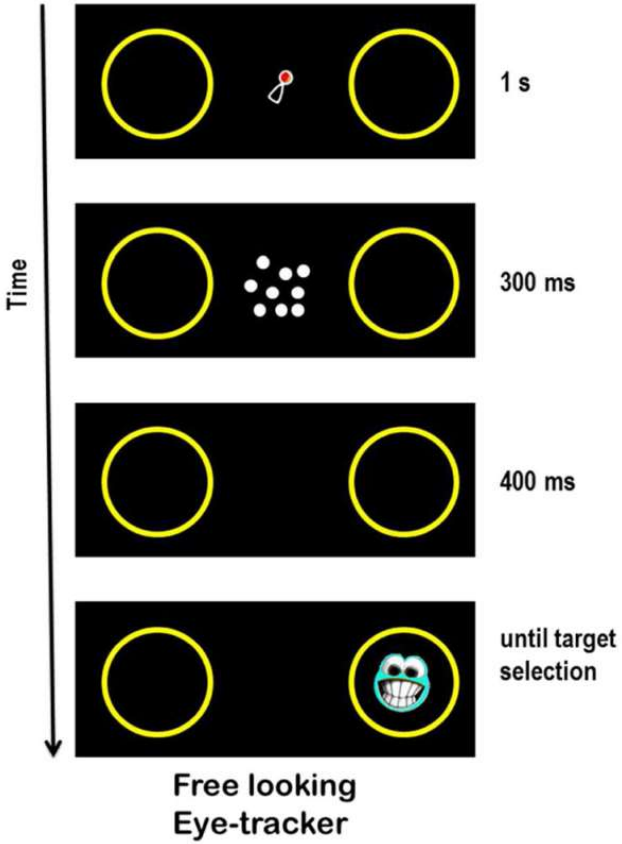
Di Giorgio, E., Lunghi, M., Rugani, R., Regolin, L., Dalla Barba, B., Vallortigara, G., & Simion, F. (2019). A mental number line in human newborns. *Developmental Science*, *22*(6), e12801. https://doi.org/10.1111/desc.12801

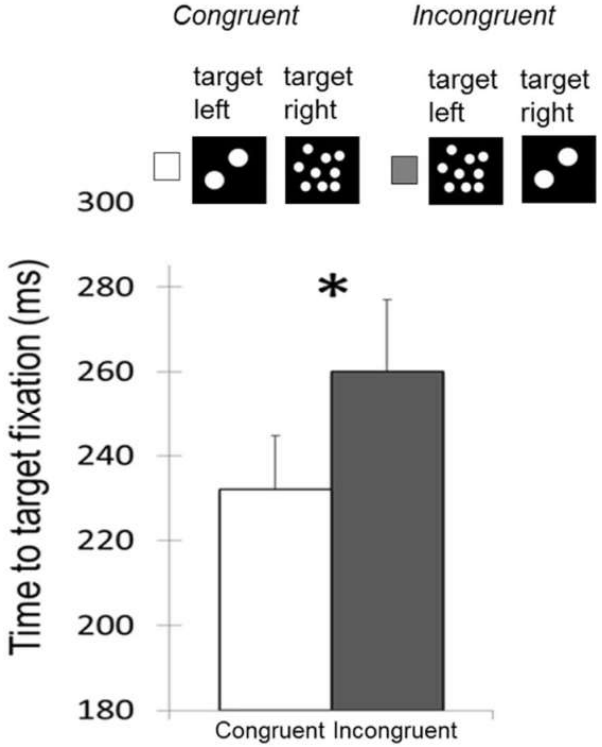
de Hevia, M. D., Veggiotti, L., Streri, A., & Bonn, C. D. (2017). At Birth, Humans Associate "Few" with Left and "Many" with Right. *Current Biology*, *27*(24), 3879-3884.e2. https://doi.org/10.1016/j.cub.2017.11.024

Bulf, H., de Hevia, M. D., & Macchi Cassia, V. (2015). Small on the left, large on the right : Numbers orient visual attention onto space in preverbal infants. *Developmental Science*, *19*(3), 394-401.

Even the direction of the association is non-arbitrary and left-to-right.

Bulf et al: in a Posner-like paradigm, an irrelevant numerical cue presented at the center of screen biases 8-month-old infants' attention to the left or the right, thus yielding faster responses to a subsequent target.





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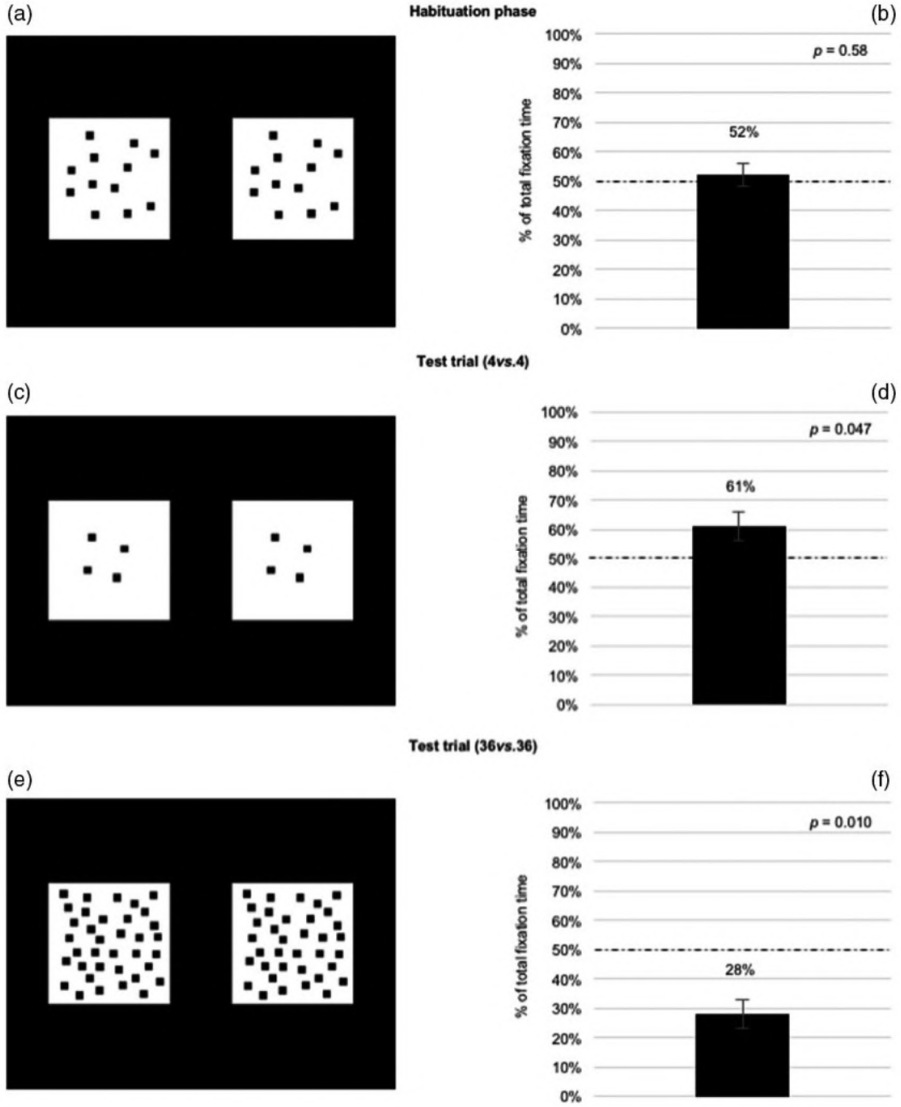
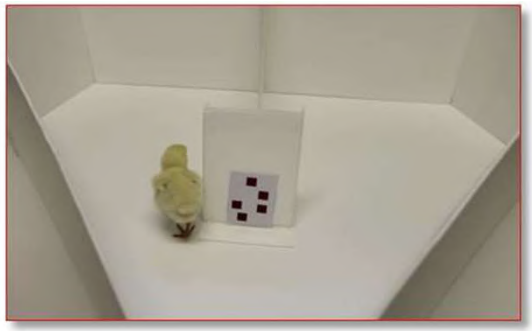
Even in newborns (mean age = 51 hours!)

Experiment 1: After habituating to a bilateral display with 12 items, neonates are presented with bilateral displays of either 4 or 36 items.
They preferentially look left for the smaller number, and right for the larger number.

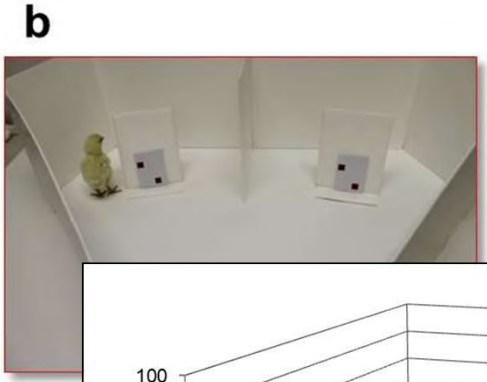
Experiment 2: Like the SNARC in adults, this effect depends on relative number size, not absolute number: the response to the display 12 depends on whether the babies were habituated with 4 or with 36.

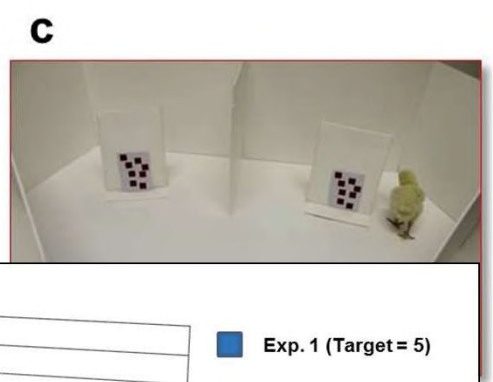


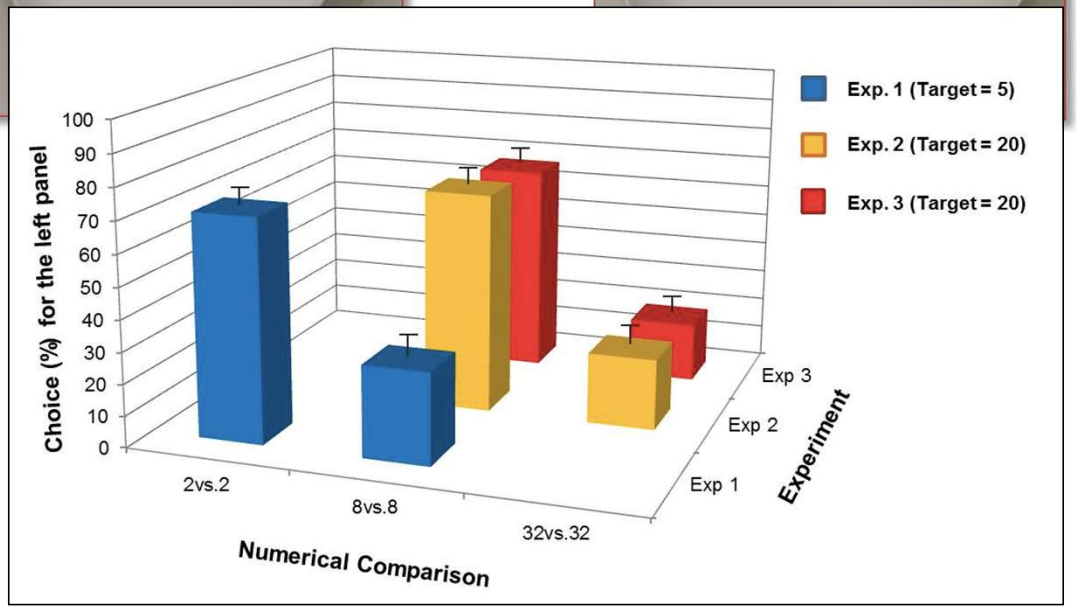
A SNARC effect in animals !



Rugani, R., Vallortigara, G., Priftis, K., & Regolin, L. (2015). Number-space mapping in the newborn chick resembles humans' mental number line. *Science*, *347*(6221), 534-536.





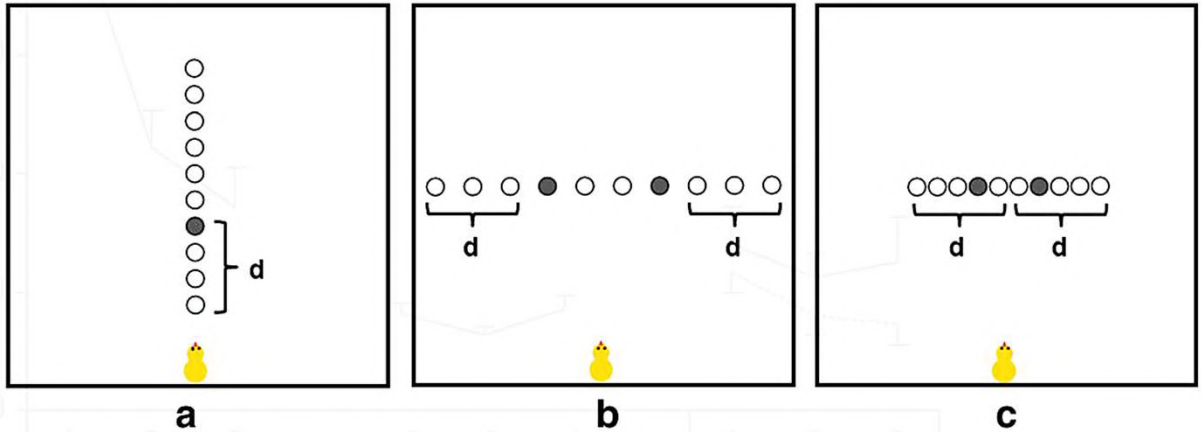


3-day-old chicks are first habituated to reach for a reward behind a central screen with a fixed number (here 5).
Then they are presented with two identical screens bearing the same number, either 2 or 8.
Result: they prefer to go left for number 2, and right for number 8!
Click to see movie 1
Experiments 2 and 3 : the central number is changed to 20. Now they prefer to go left for 8, and right for 32!

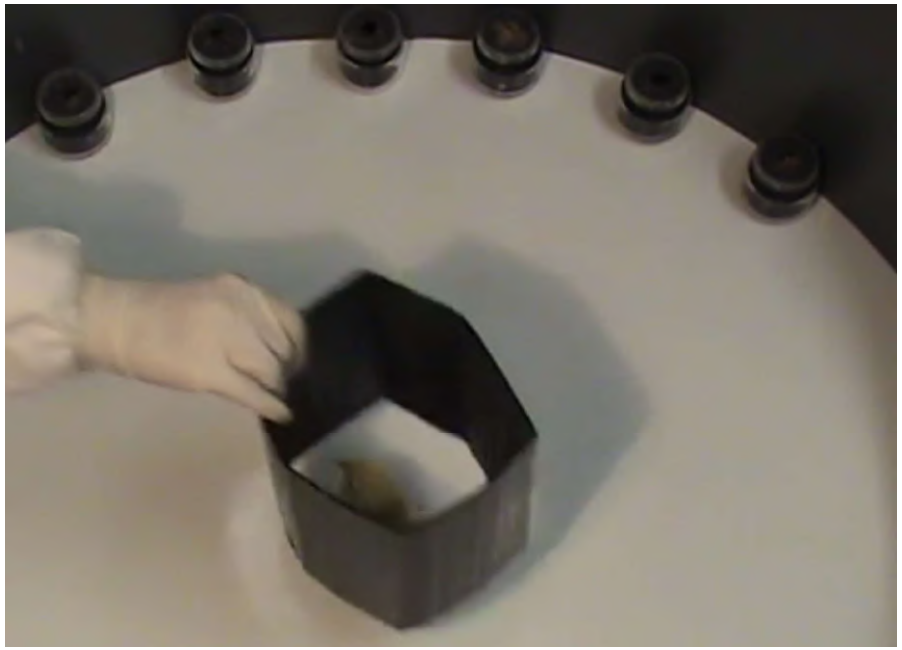
Similar experiments in monkeys: Rugani, R., Platt, M. L., Zhang, Y., & Brannon, E. M. (2024). Magnitude shifts spatial attention from left to right in rhesus monkeys as in the human mental number line. *iScience*, *27*(2), 108866. https://doi.org/10.1016/j.isci.2024.108866

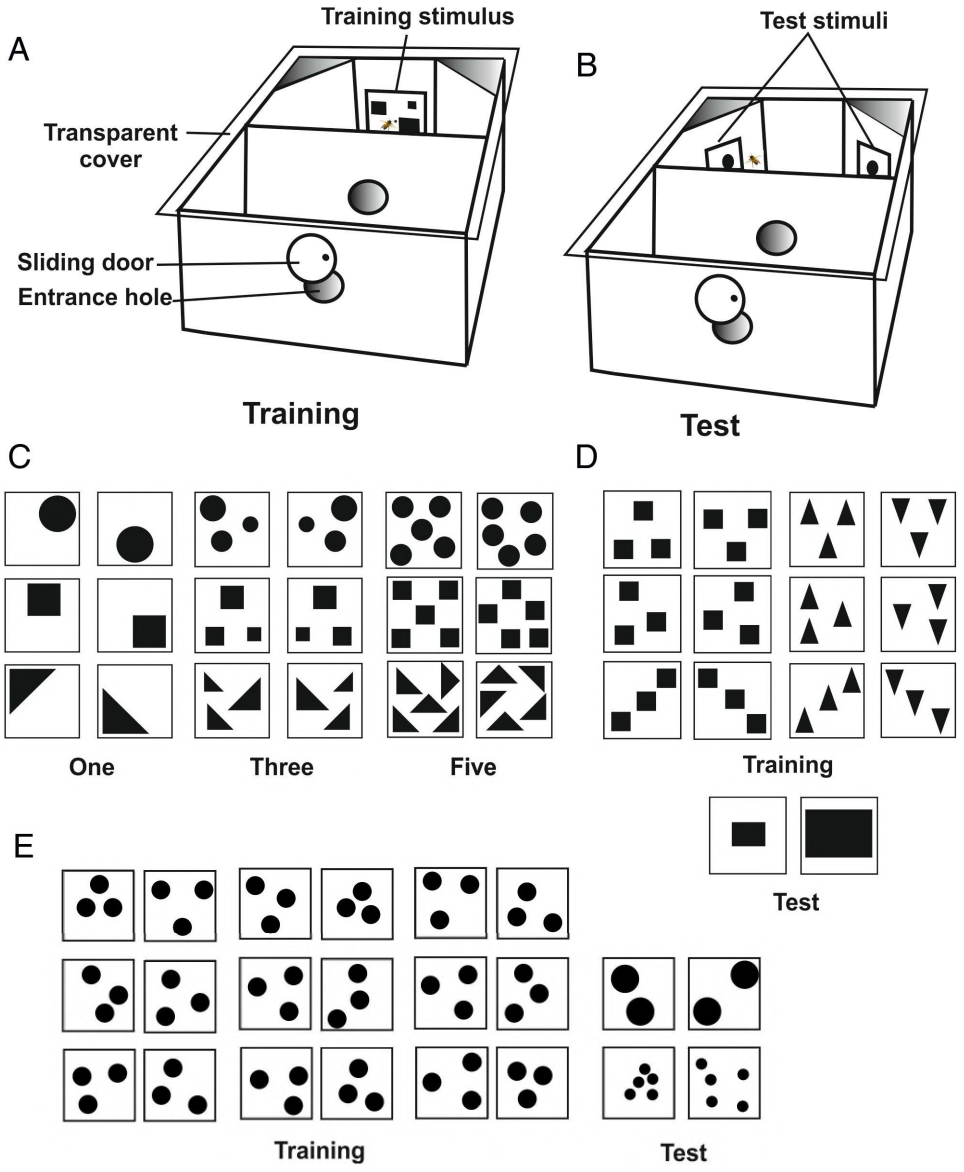
Chicks “count” from left to right

Rugani, R., Regolin, L., & Vallortigara, G. (2007). Rudimental numerical competence in 5-day-old domestic chicks (Gallus gallus) : Identification of ordinal position. *J Exp Psychol Anim Behav Process*, *33*(1), 21-31. https://doi.org/10.1037/0097-7403.33.1.21



Chicks trained to pick at the 4th location in a sagital row of items generalize to a fronto-parallel arrangement of the same items.
Crucially, they do so from left to right!
Further experiments show that behavior is driven by ordinal number rather by distance or spacing.





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And even in bees:

Giurfa, M., Marcout, C., Hilpert, P., Thevenot, C., & Rugani, R. (2022). An insect brain organizes numbers on a left-to-right mental number line. *Proceedings of the National Academy of Sciences*, *119*(44), e2203584119. https://doi.org/10.1073/pnas.2203584119

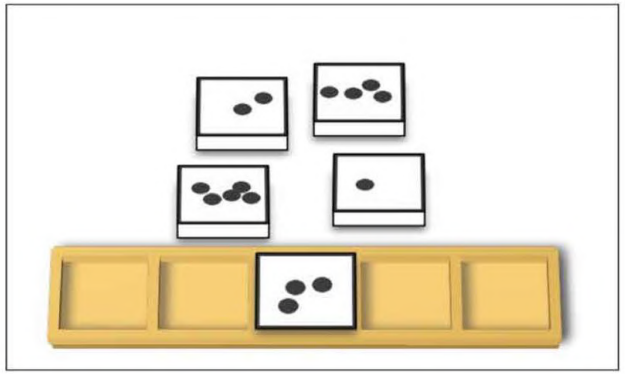
No clear explanation yet – but this may relate to

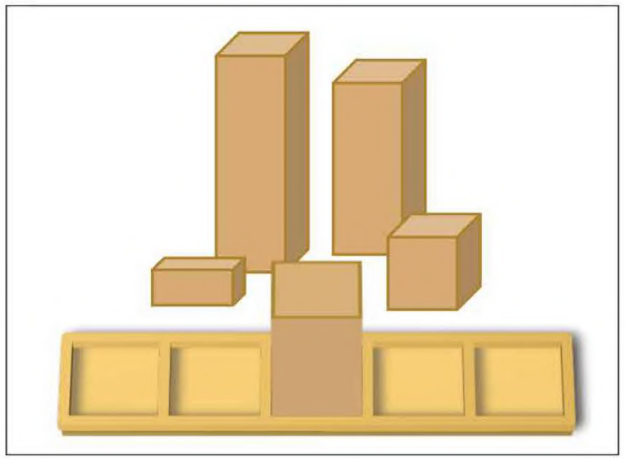
- a right-hemispheric attentional bias
- or larger numbers may be interpreted as a positive experience, hence (?) activating more the left hemisphere.

No correspondence between number and space in uneducated adults ?

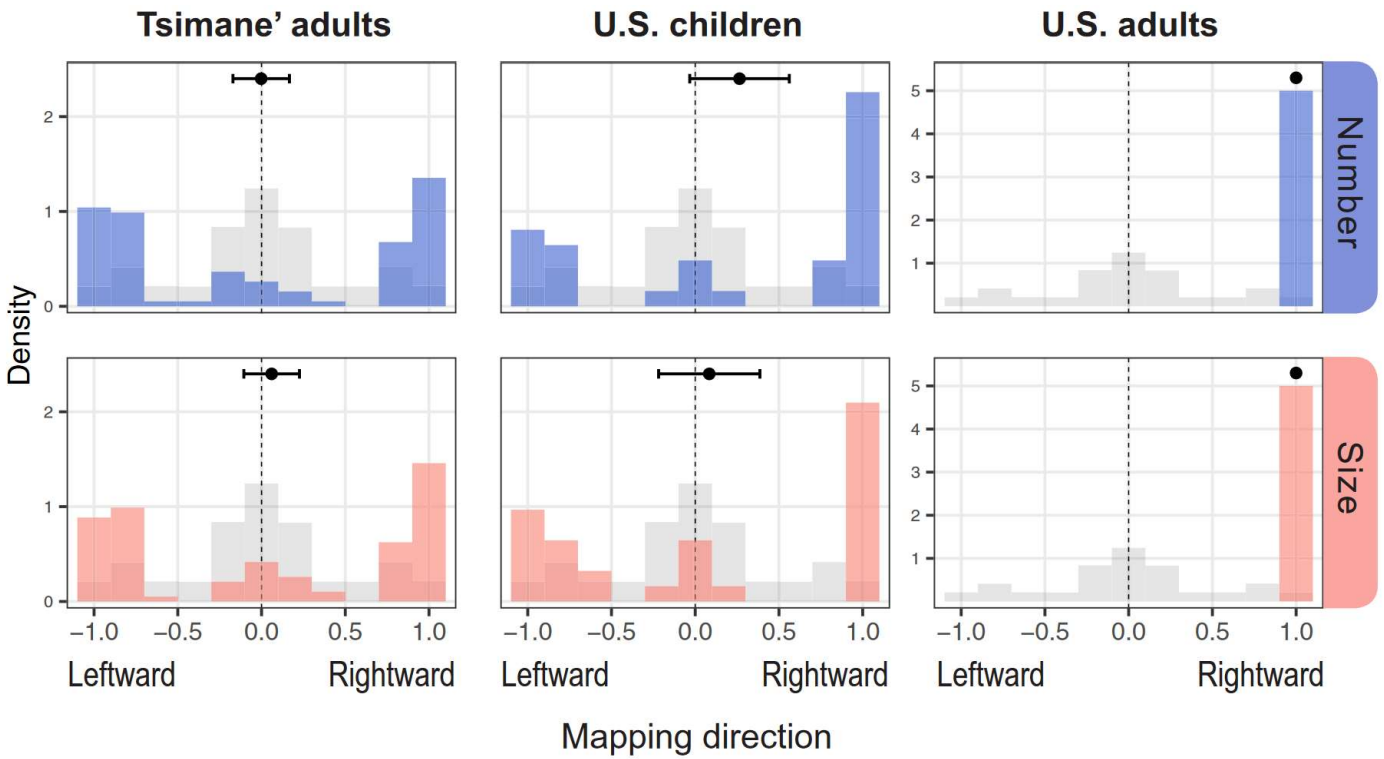
Pitt, B., Ferrigno, S., Cantlon, J. F., Casasanto, D., Gibson, E., & Piantadosi, S. T. (2021). Spatial concepts of number, size, and time in an indigenous culture. *Science Advances*, *7*(33), eabg4141. https://doi.org/10.1126/sciadv.abg4141

Pitt, B., Casasanto, D., & Piantadosi, S. T. (2023). No clear evidence for an innate left-to-right mental number line. *Proceedings of the National Academy of Sciences*, *120*(28), e2306099120. https://doi.org/10.1073/pnas.2306099120





Adult Tsimane were asked to order number, size, and time stimuli on a left-right axis.



A crucial distinction between explicit and implicit number-space tasks

Eccher, E., Josserand, M., Caparos, S., Boissin, E., Buiatti, M., Piazza, M., & Vallortigara, G. (2025). A left-to-right bias in number-space mapping across ages and cultures. Nature Communications, 16(1), 495. https://doi.org/10.1038/s41467-024-55685-x

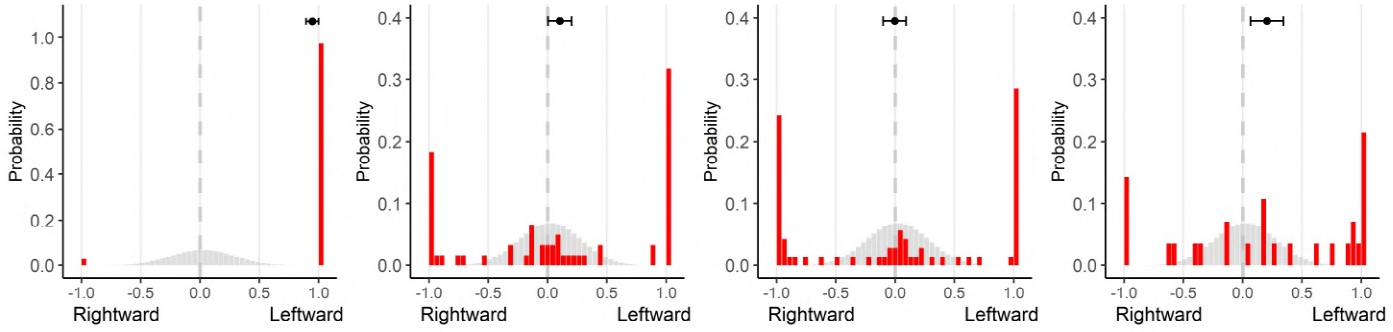
47 Italian Adults
- Mean age 21.67 ± 2,3 years
- 36 Female
- Highly educated

133 Himba Adults (2021 & 2022)
- Mean age: 30.52 ± 11.13 years.
- 65 Female
- Only oral culture

43 Italian Preschooler
- Mean age: 5.10 ± 0.93 years
- 21 Female
- 2nd year of Kindergarten

Explicit task:

"Look at the cards [with 1-10 dots] and place them in order on the table"



Italian adults | Himba 2021 | Himba 2022 | Italian preschoolers

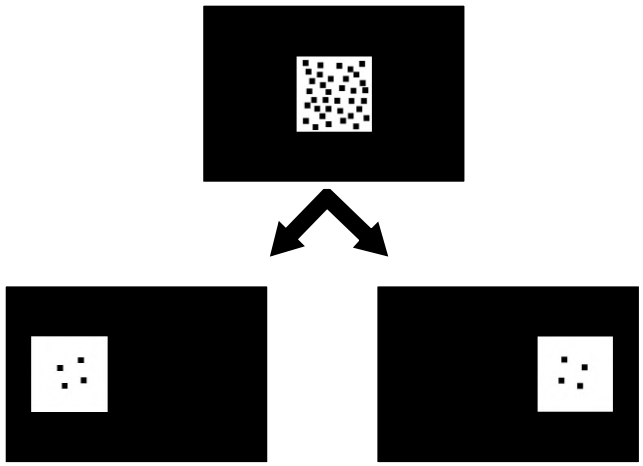
All distributions (red) differ from chance distribution (grey), but only Italian adults show a Left-to-Right bias in their explicit behaviour

A crucial distinction between explicit and implicit number-space tasks

Eccher et al (2025) *Nature Communications*

Implicit task: similar to animals !
On each trial, a reference number is presented,
then a target number can appear left or right, and it can be smaller or larger.
Task = go / nogo number comparison

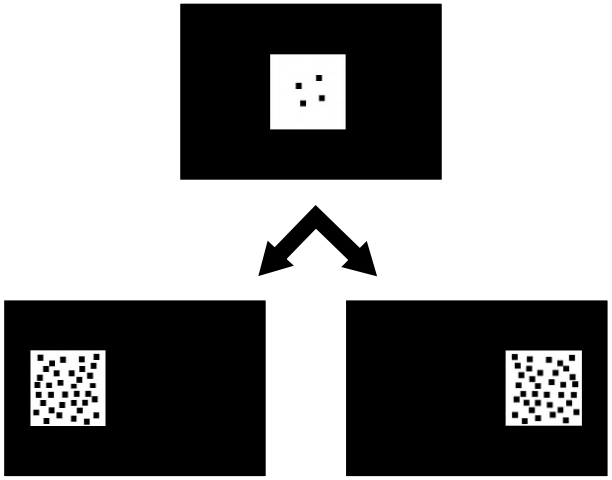
"Press when less"



Congruent

Incongruent

"Press when more"



Incongruent

Congruent

A crucial distinction between explicit and implicit number-space tasks

Eccher et al (2025) *Nature Communications*

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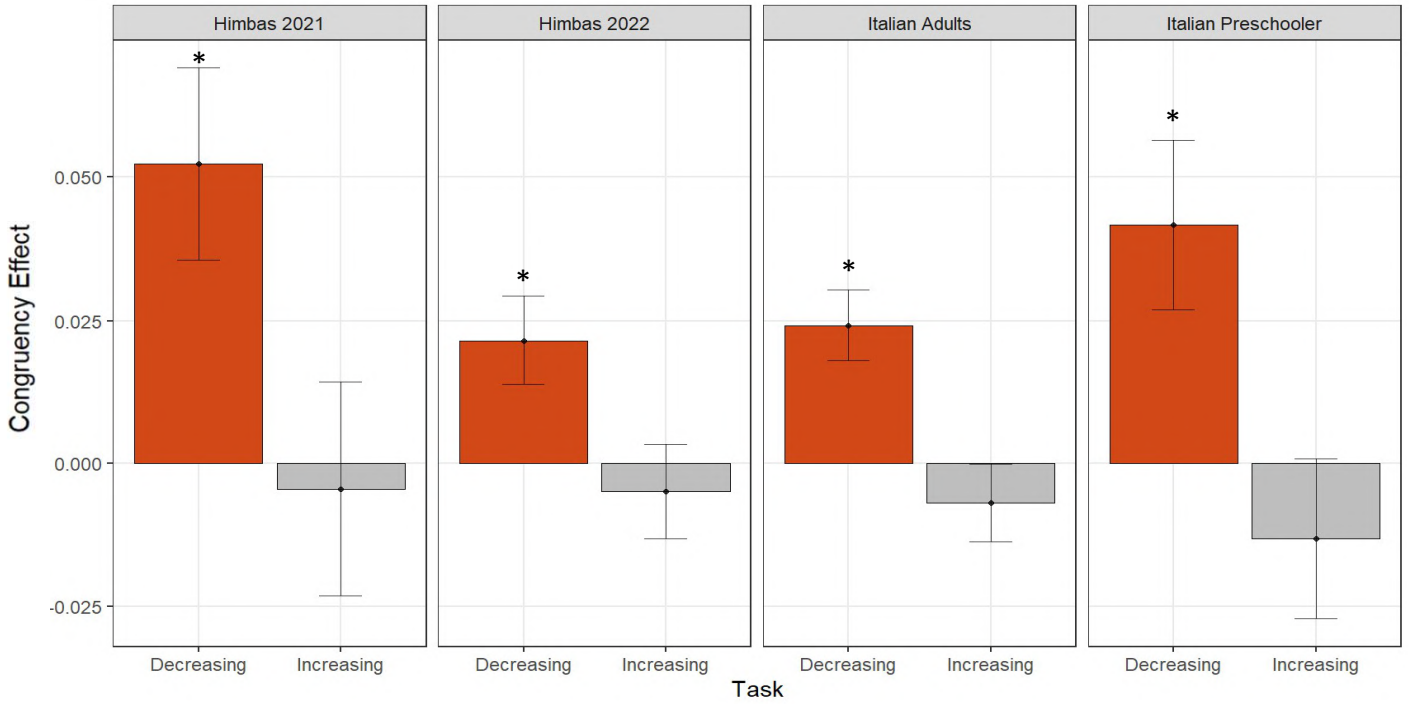


Italian adults | Himba 2021 | Himba 2022 | Italian preschoolers

Results :
In their response times, all subjects show a congruency effect, with faster RTs for smaller numbers appearing on the left side than on the right side.

Conclusions:

- The **intuition of a number-to-space mapping** is present in all humans and in many animals.
- It goes **from left to right**.

The shape of the SNARC: From logarithmic to linear

Siegler & Opfer, 2003; Siegler & Booth, 2004

Number-Space mapping task: « Please point to where number x should fall » (or conversely : which number goes there?)

A major change occurs during mathematical education: a switch from a logarithmic to a linear understanding of number

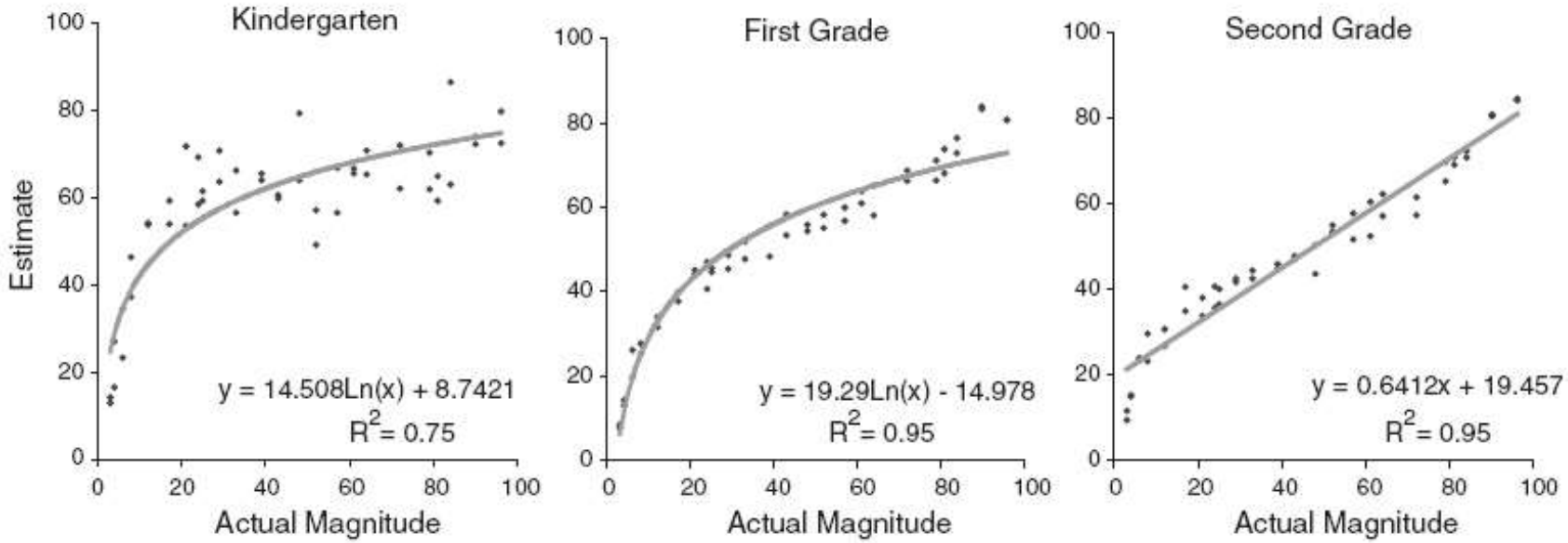


Figure 2. Progression from logarithmic pattern of median estimates among kindergartners (left panel) to linear pattern of estimates among second graders (right panel) in Experiment.

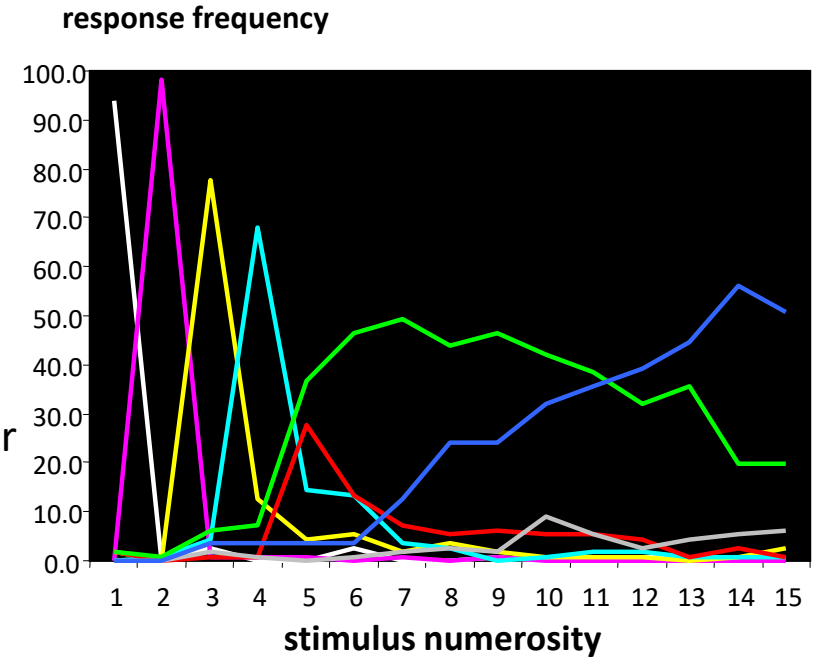
Can the Munduruku understand number-space mappings ? If so, are they logarithmic?

Pica, Lemer, Izard, & Dehaene, Science, 2004

- pug ma = one
- xep xep = two
- ebapug = three
- ebadipdip = four
- pug põgbi = one hand
- xep xep põgbi = two hands
- adesu/ade gu = some, not many
- ade/ade ma = many, really many

A reduced lexicon of number words

Munduruku number words refer to approximate numerosity

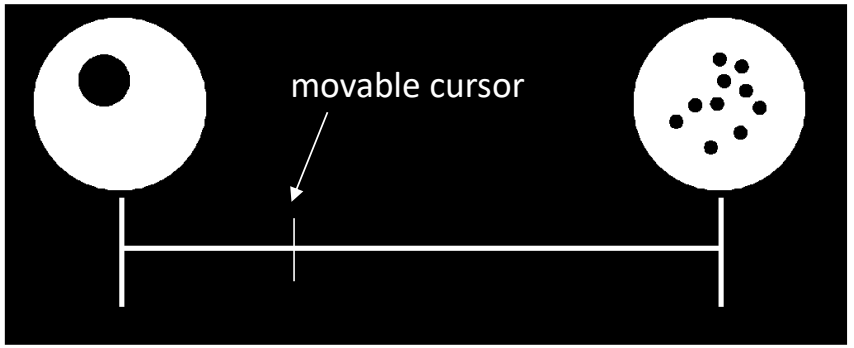


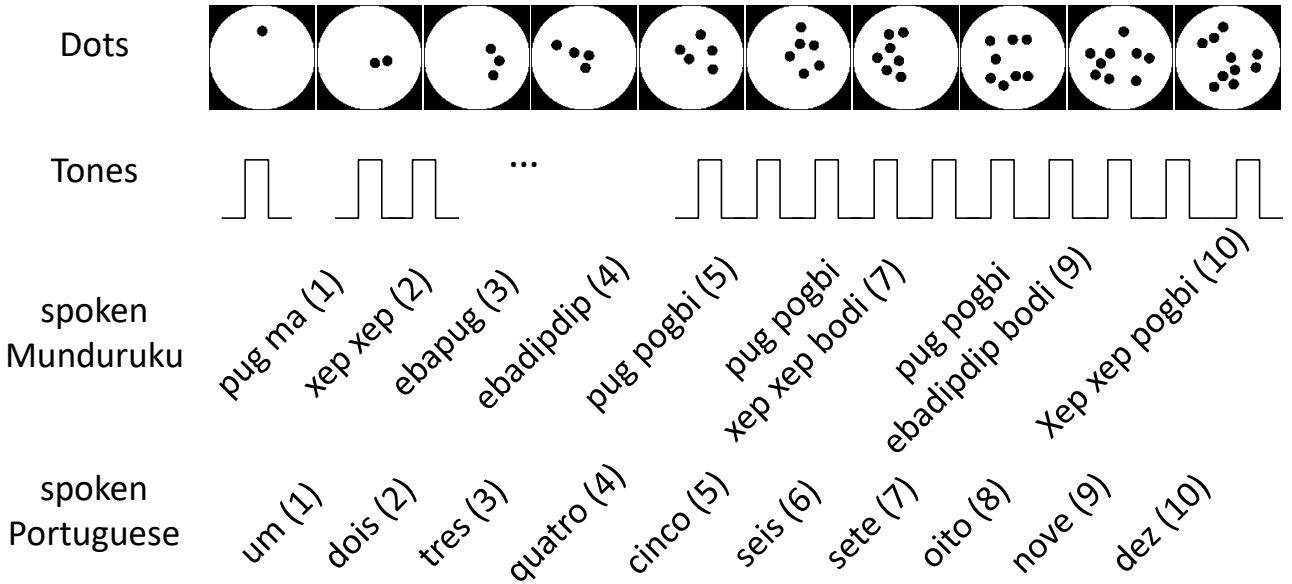
Munduruku adults and children can do approximate arithmetic with non-verbal numerosities (e.g. 40+30 is larger than 50) but not exact arithmetic (e.g. 7-6=1)

Number-Space mappings in the Munduruku

Dehaene, Izard, Spelke and Pica, Science, 2008

Munduruku children and adults were asked to point to the location corresponding to a certain number.





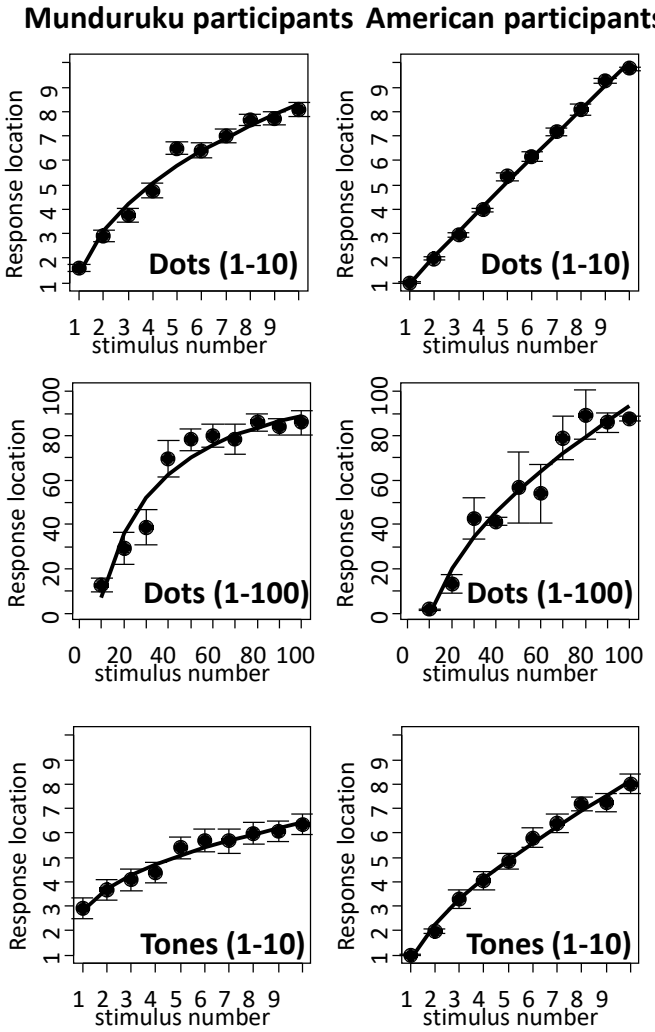
Logarithmic Number-Space mapping in Munduruku adults

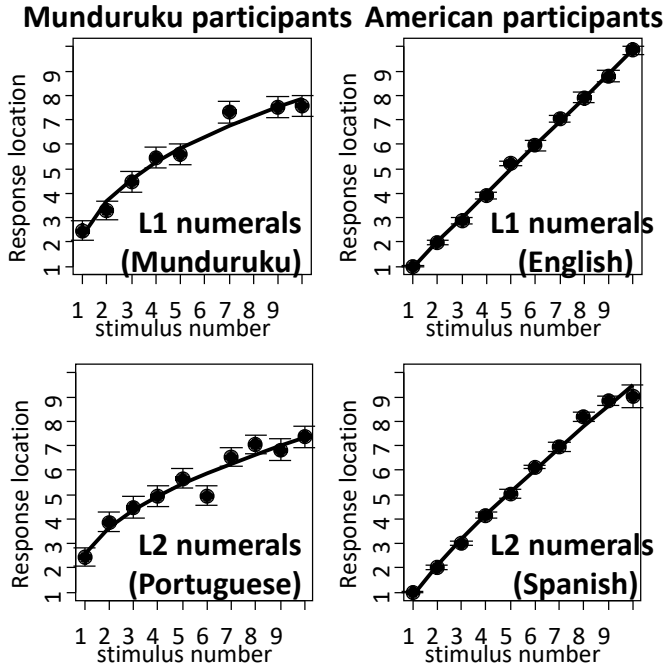
Dehaene, Izard, Spelke and Pica, Science 2008

Munduruku children and adults show a compressive mapping

- For dot patterns or series of 1-10 tones

- For Munduruku words and even for Portuguese numerals





Debates on the interpretation of the logarithmic effect

Asmuth, J., Morson, E. M., & Rips, L. J. (2018). Children's Understanding of the Natural Numbers' Structure. *Cognitive Science*, *42*(6), 1945-1973. https://doi.org/10.1111/cogs.12615

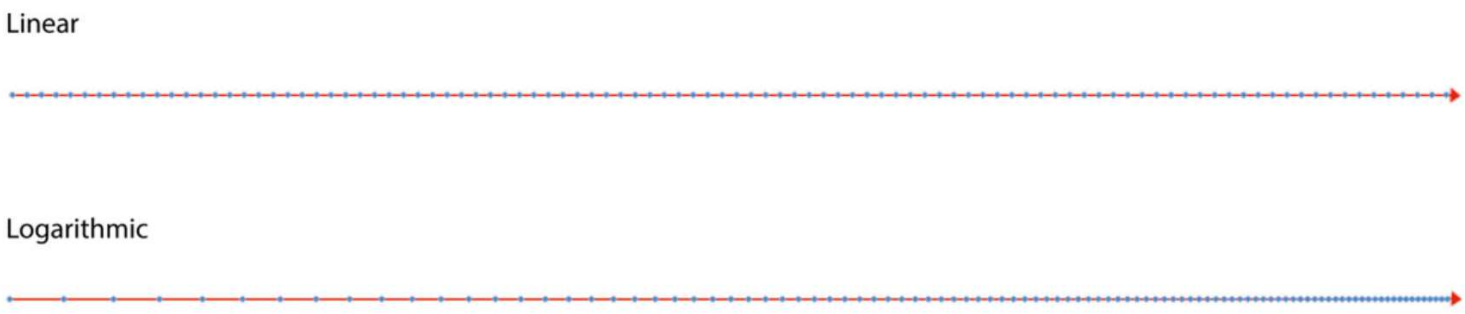
A compressive, logarithmic representation of numbers, whereby large numbers are closer together than small numbers, is attested by many experiments, e.g.

- Behavioral data on the perception and comparison of non-symbolic numerosities
- The neural representation of non-symbolic numerosity, in both animals and humans
- The perception of random sequences (Viarouge, Hubbard, Dehaene & Sackur, 2010):
  e.g. in the interval 0-750, the average "most random" sequence is 3, 26, 65, 118, 186, 270, 368, 480, 608, 750
- The representation of prices

Two things are debated, however:

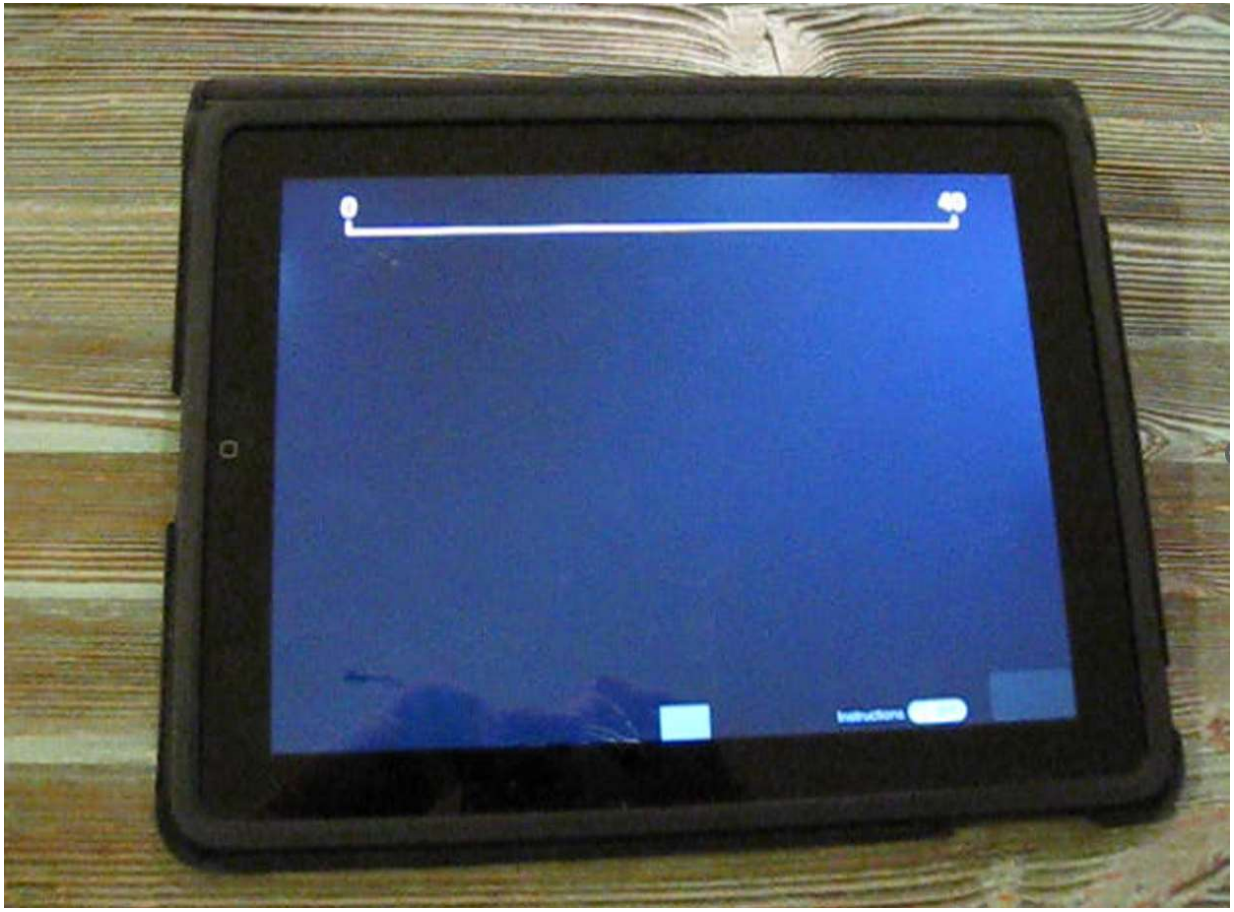
1. Could the same data be explained by a **linear representation of number with scalar variability**?
   In most cases, yes, but the neural data directly suggests an internal compressive logarithmic representation.
2. Could alternative explanations account for the compressed **number-to-space mapping** ?

One possibility is that children group together numbers that are large, infrequent, unfamiliar.
Indeed, when simply asked which line provides the best representation of numbers 1-100, children prefer the linear one.



Decomposing the number-space task

Dror Dotan programmed the ipad to digitize the finger trajectory while subjects perform the number-to-line task (with two-digit numbers between 0 and 40)

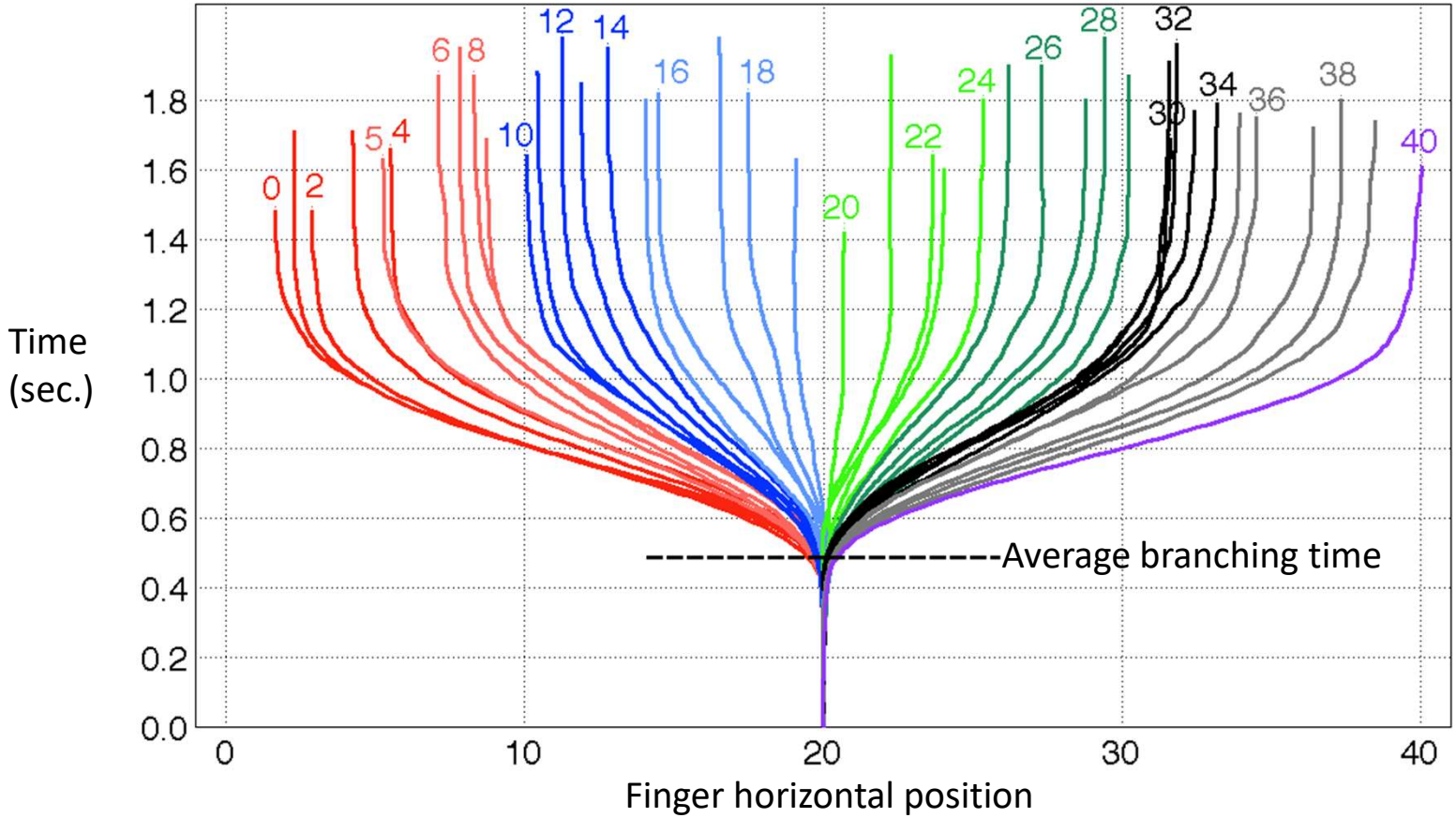


Dror Dotan

- Dotan, D., & Dehaene, S. (2013). How do we convert a number into a finger trajectory? *Cognition*, *129*(3), 512-529. https://doi.org/10.1016/j.cognition.2013.07.007
- Dotan, D., & Dehaene, S. (2016). On the origins of logarithmic number-to-position mapping. *Psychological review*, *123*(6), 637.
- Dotan, D., & Dehaene, S. (2020). Parallel and serial processes in number-to-quantity conversion. *Cognition*, *204*, 104387. https://doi.org/10.1016/j.cognition.2020.104387
- Dotan, D., Pinheiro-Chagas, P., Al Roumi, F., & Dehaene, S. (2019). Track It to Crack It : Dissecting Processing Stages with Finger Tracking. *Trends in Cognitive Sciences*, *23*(12), 1058-1070. https://doi.org/10.1016/j.tics.2019.10.002

Decomposing the number-space task

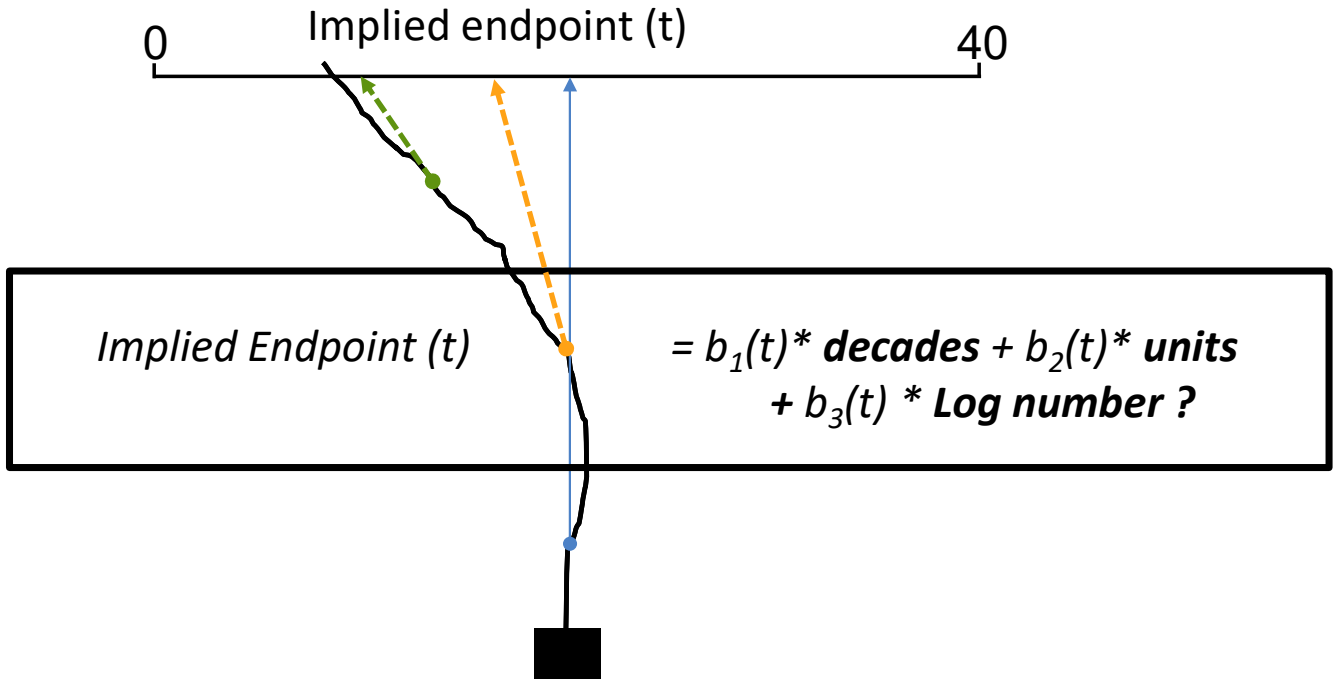
Plotting the x coordinate as a function of time reveals the dynamics of number processing.



Decomposing the number-space task

1. Implied endpoint indicates where the subject is aiming at each moment *t*

2. Multiple regression determines which variables govern this intended response.



Dotan & Dehaene, Cognition (2013)

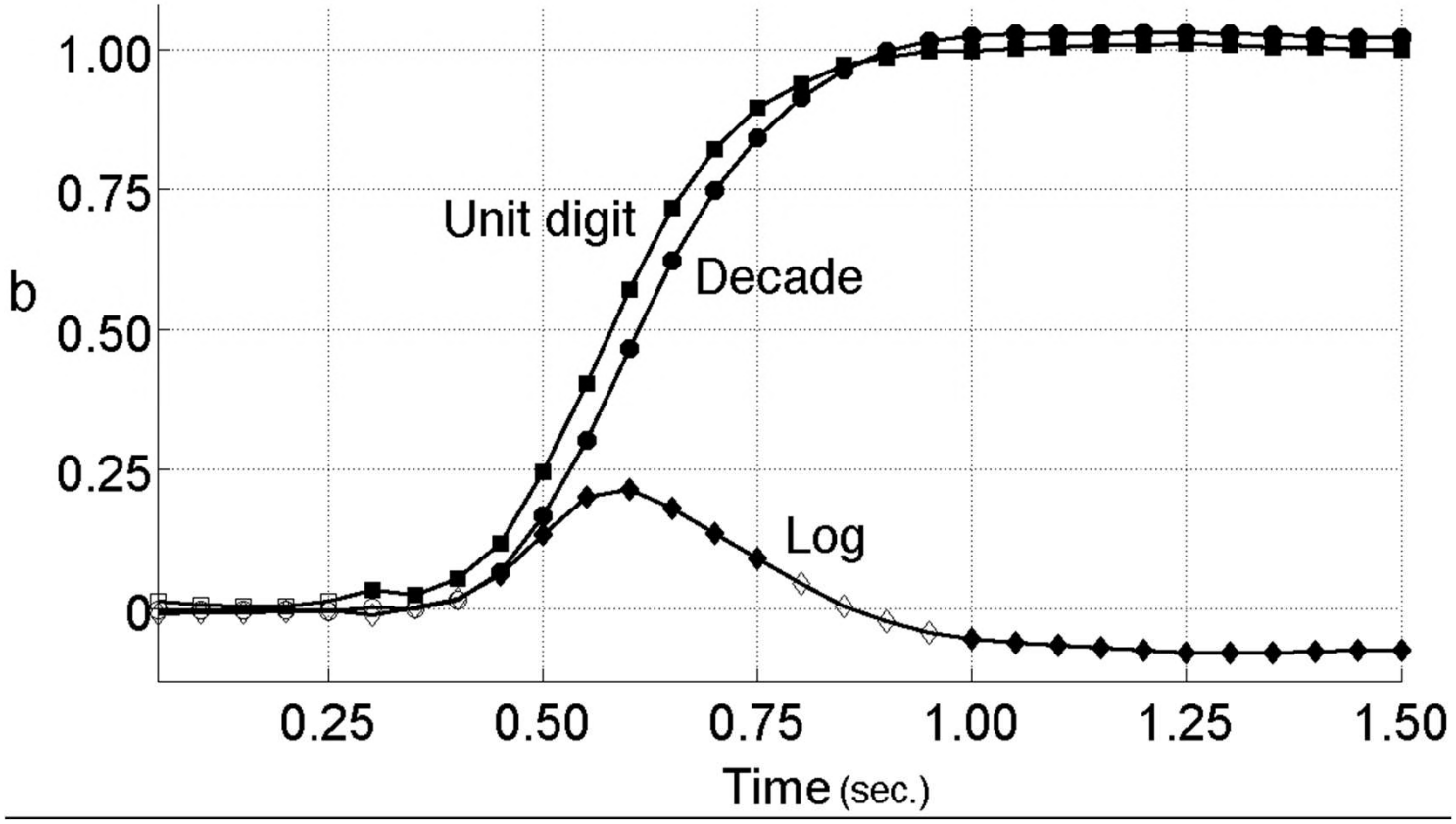
The number-space task in normal adults

Units and decades are processed nearly simultaneously

→ holistic apprehension of the quantity – or even enhancement of units

There is a transient effect of the log quantity

→ a dormant compressive representation in educated adults?



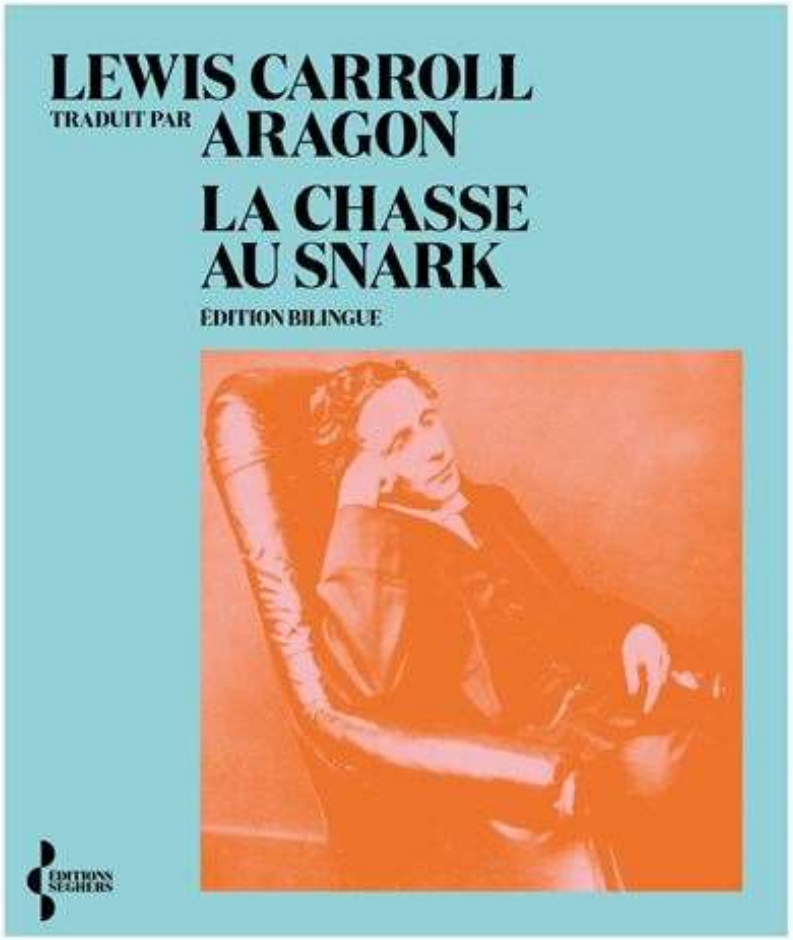
But… the effet could also arise if larger numbers were responded more slowly. A positive log effect would indicate that the curve rises faster for small numbers than for large ones.

And indeed – when data are aligned on the moment of first detection of a movement to the side, rather than on stimulus onset, then the log effect disappears (Dotan & Dehaene 2016).

Conclusions:

- With number symbols, adult subjects behave **linearly**, yet with a increasing delay for larger numbers.
- With non-symbolic numbers, the data is best captured by a logarithmic effect.
- with symbols in children, the debate is unsettled.

Conclusions



The mapping of numerical quantities onto space is an essential component of most graphics.

Fortunately, it is relatively **intuitive**:

- Many cultures invented measurement of space by numbers
- Intuitions of number-space mapping are present in
  - Remote cultures
  - Infants
  - Even non-human animals (chicks, monkeys, bees)

Even the **directionality** of the mapping is intuitive: a left-to-right bias already exists in non-human animals and infants.

The **scale** can be initially compressive, and move to linear with education.

This mapping is probably due to the tight relations between **neural representations of number and space in the parietal lobe**.

It **must** be tightly respected in graphics, for both efficiency and ethics reasons.