Cours 2024-2025:

La perception des graphiques: un nouvel exemple de recyclage neuronal

The perception of graphics : a new example of neuronal recycling

Stanislas Dehaene
Chaire de Psychologie Cognitive Expérimentale

Cours n°4 bis

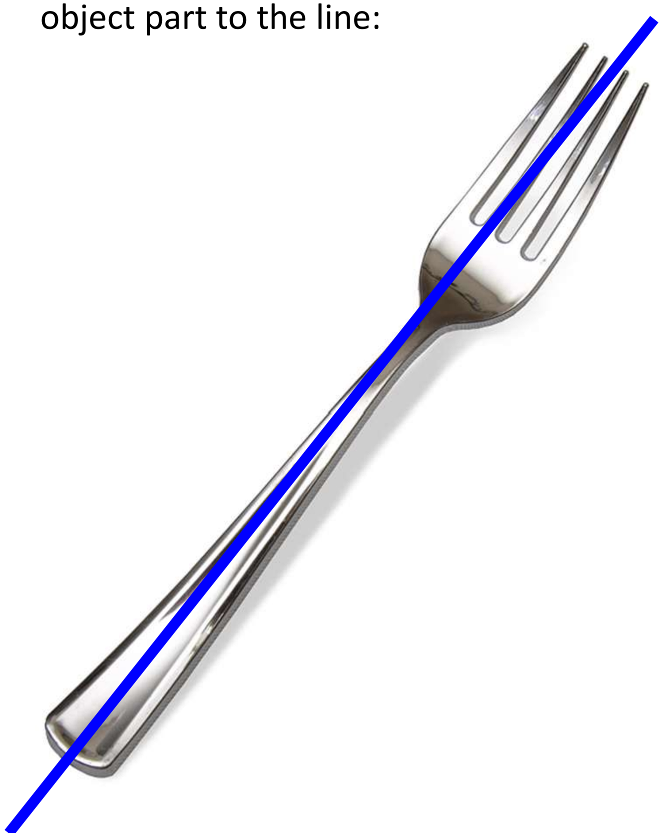
La perception des tendances et des courbes

Perceiving trends and curves

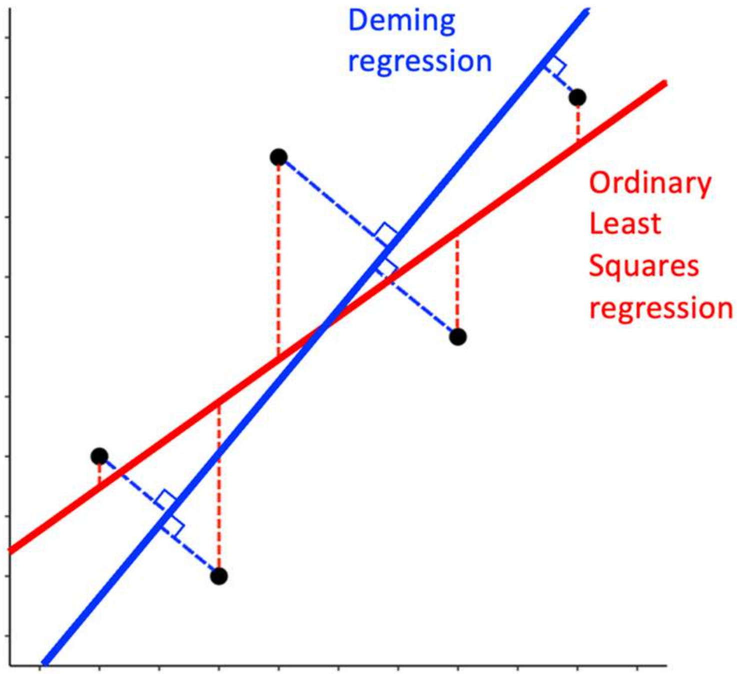
Prediction 2 : The principal axis of an object corresponds to Deming regression, not ordinary least squares

Ciccione, L., & Dehaene, S. (2021). Can humans perform mental regression on a graph? Accuracy and bias in the perception of scatterplots. *Cognitive Psychology*

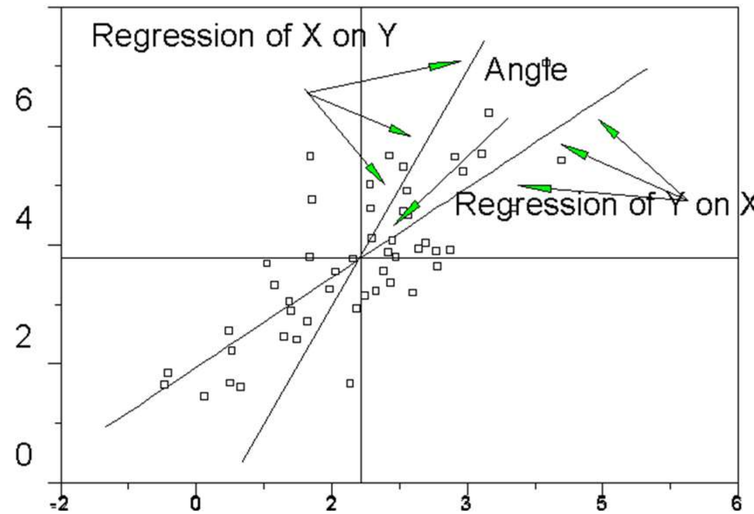
The principal axis of an object is defined as the line whose orientation minimizes the sum of the square distances of each object part to the line:



For a scatter plot, this corresponds to Deming regression, which differs from Ordinary Least Squares. The latter assumes y = a x + b + noise, and therefore minimizes the vertical distance to the line.



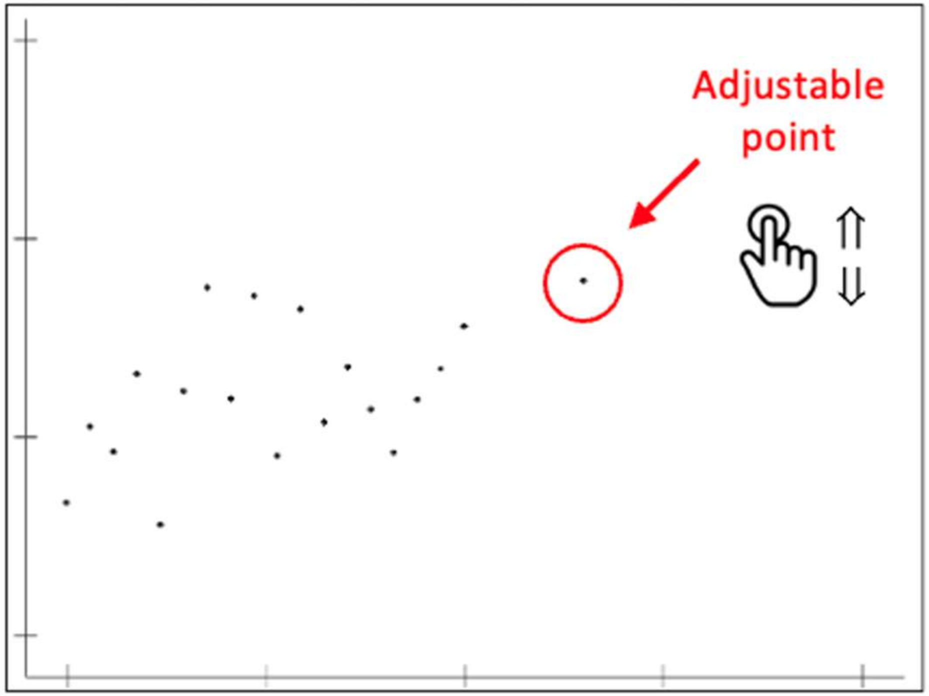
OLS regression provides unbiased estimates of the slope a and the intercept b, but it has some counterintuitive properties.
Notably, there are **two** regression lines, one for regressing Y on X, and one for regressing X on Y.
The regression coefficient r is the same for Deming and OLS.

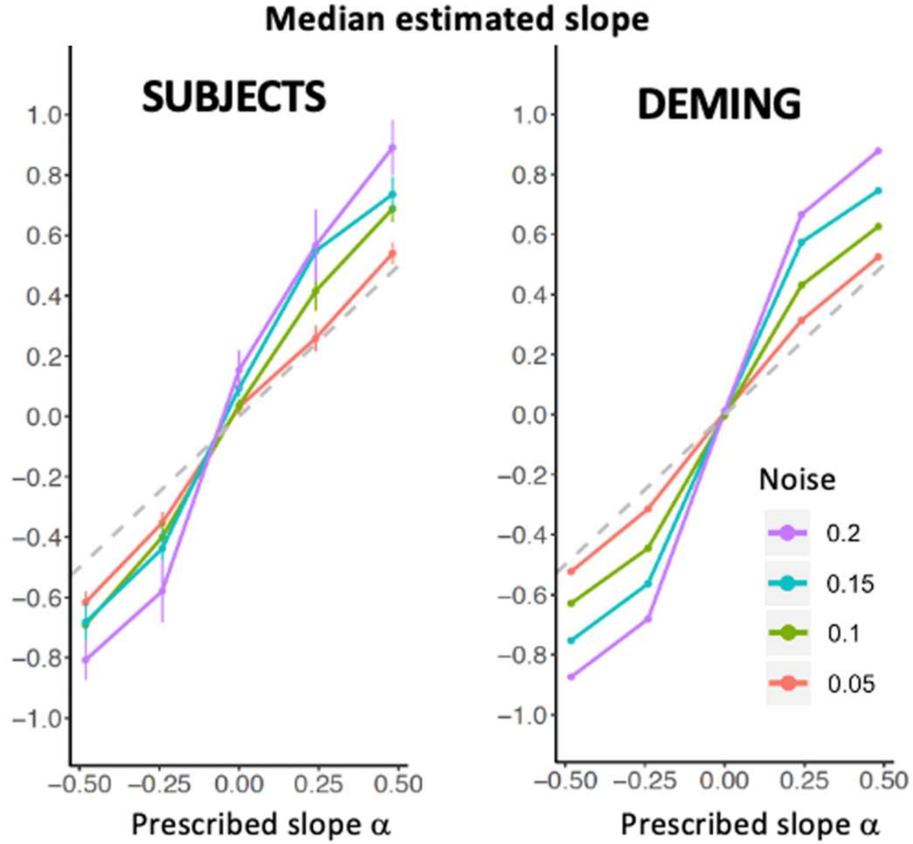


Extrapolation task

Ciccione, L., & Dehaene, S. (2021). Can humans perform mental regression on a graph? Accuracy and bias in the perception of scatterplots. *Cognitive Psychology*

Those results were replicated with a second method, a more implicit task where subjects extrapolated a regression by placing a dot.

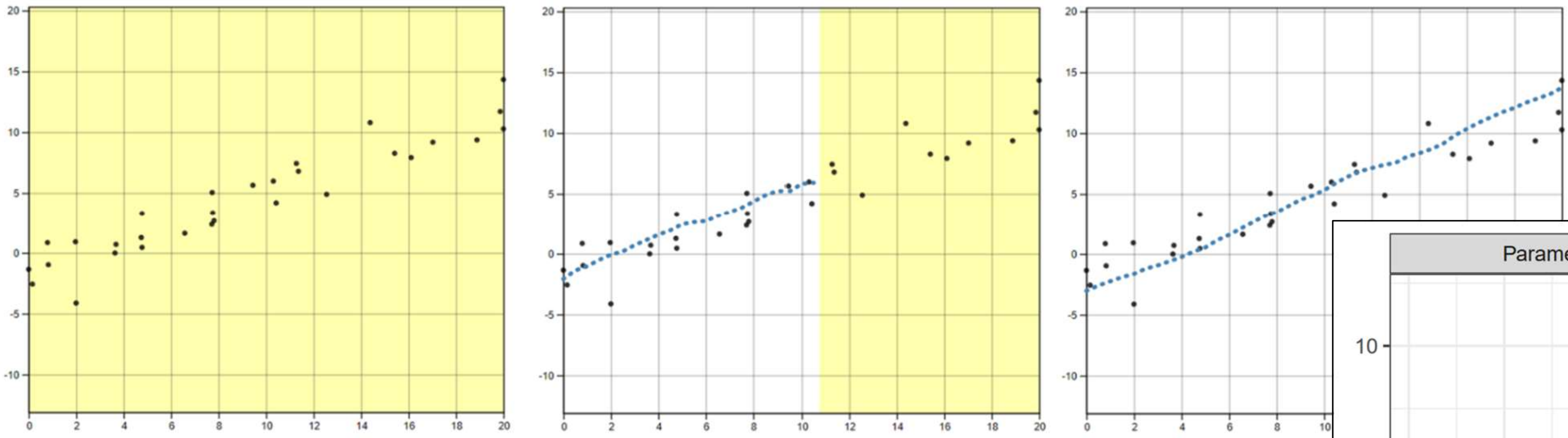


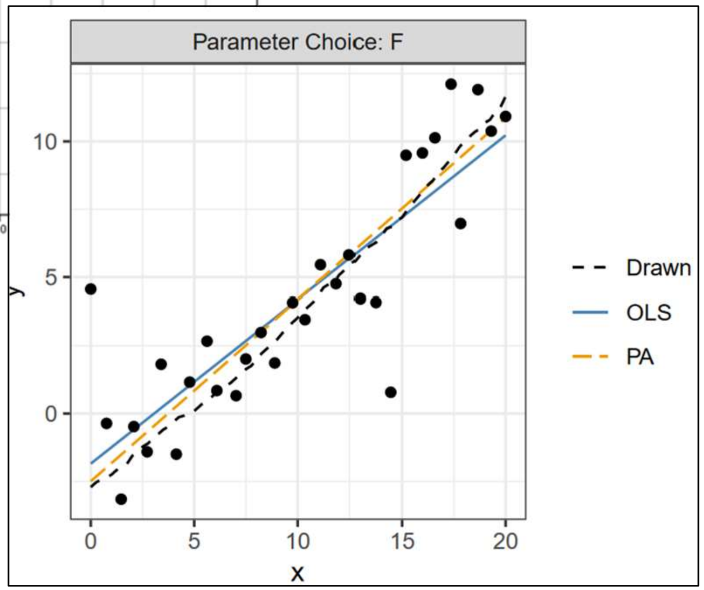


Similar results were obtained by others

Mosteller, F., Siegel, A. F., Trapido, E., & Youtz, C. (1981). Eye Fitting Straight Lines. *The American Statistician*, *35*(3), 150-152. https://doi.org/10.1080/00031305.1981.10479335
Robinson, E. A., Howard, R., & VanderPlas, S. (2023). Eye Fitting Straight Lines in the Modern Era. *Journal of Computational and Graphical Statistics*, *32*(4), 1537-1544. https://doi.org/10.1080/10618600.2022.2140668

The “you draw it” task : on a computer, using a mouse, subjects draw the trend they see in a graphic (from left to right).

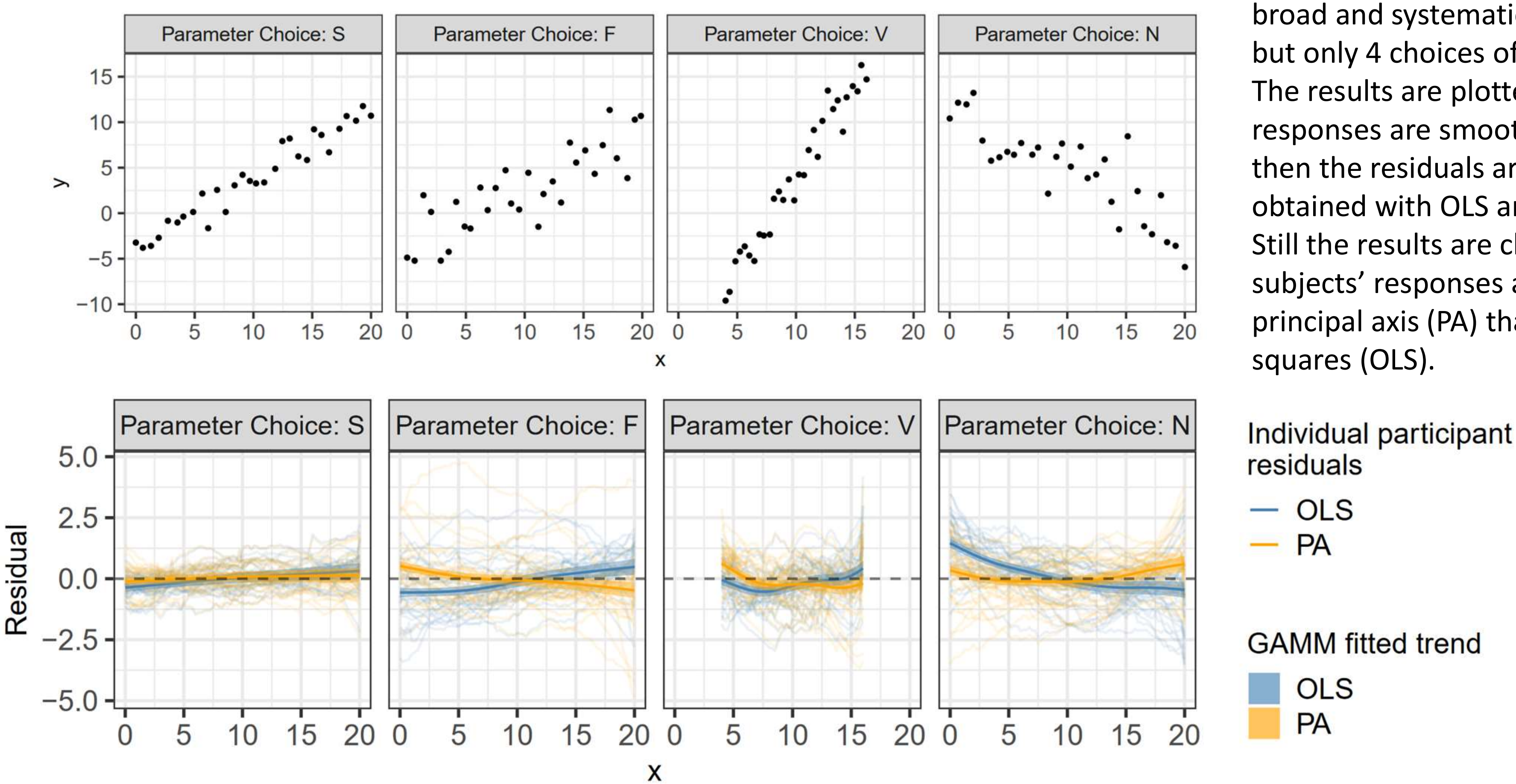




Example of what one subject drew, compared to the lines obtained by ordinary least squares (OLS) and principal axis (PA = Deming regression).

Similar results were obtained by others

Robinson, E. A., Howard, R., & VanderPlas, S. (2023). Eye Fitting Straight Lines in the Modern Era. *Journal of Computational and Graphical Statistics*, *32*(4), 1537-1544. https://doi.org/10.1080/10618600.2022.2140668



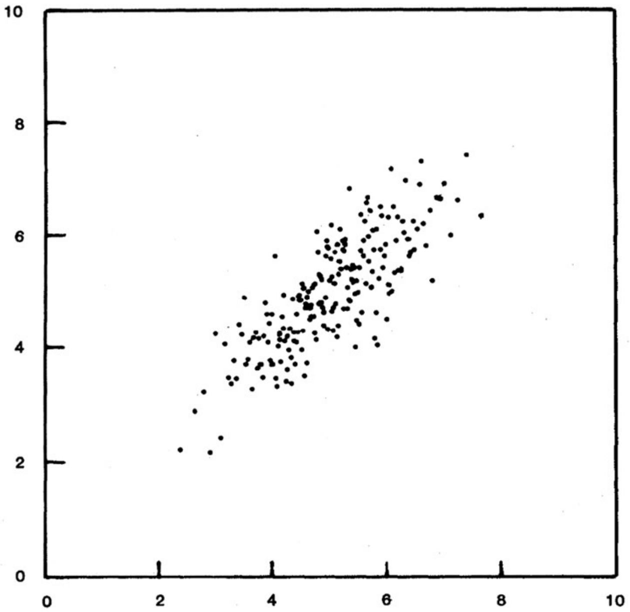
Contrary to us, the authors do not explore a broad and systematic range of parameters, but only 4 choices of linear relationships.

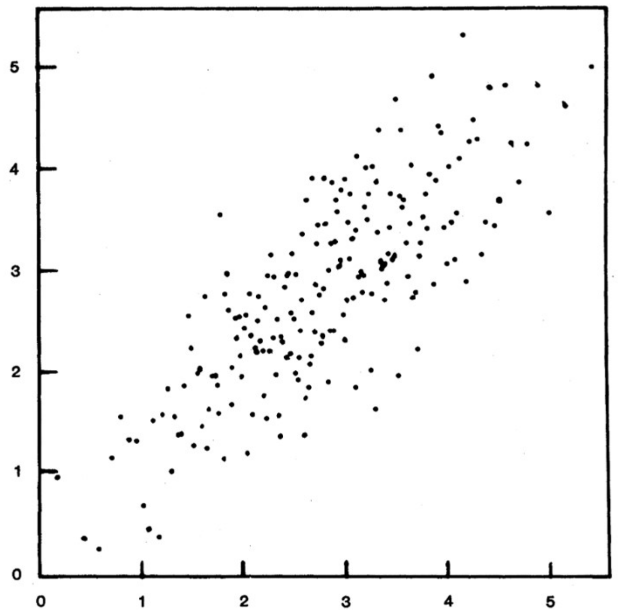
The results are plotted in a funny way: first the responses are smoothed with a spline, and then the residuals are compared to those obtained with OLS and with PA.

Still the results are clear: in all four cases, the subjects' responses are better fit with the principal axis (PA) than with ordinary least squares (OLS).

Another result compatible with object perception : Biases in judging the strength of an association from a scatterplot

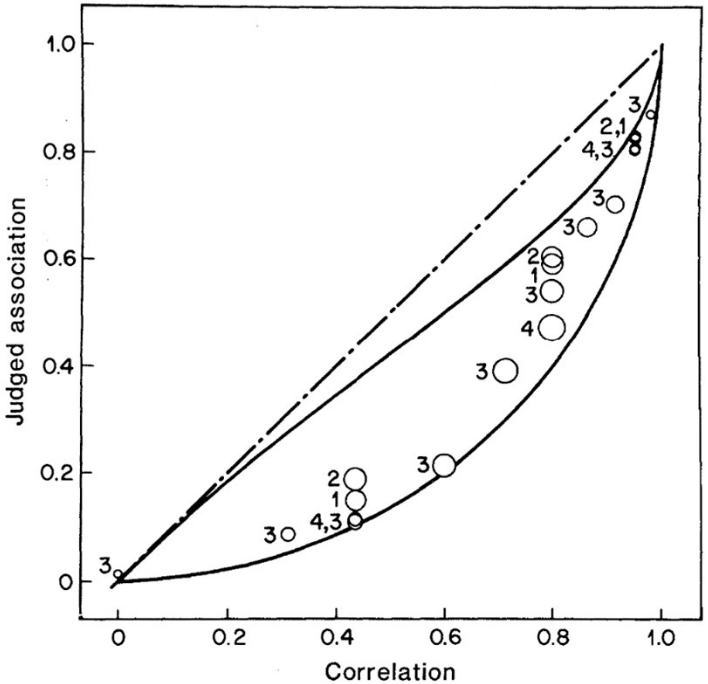
Cleveland, W. S., Diaconis, P., & McGill, R. (1982). Variables on Scatterplots Look More Highly Correlated When the Scales Are Increased. *Science*, *216*(4550), 1138-1141. https://doi.org/10.1126/science.216.4550.1138





Two results:

- The subjective association grows non-linearly with r (we will return to this point)
- But the association is also judged stronger when the scale is increased, so that the data seems more "compact".



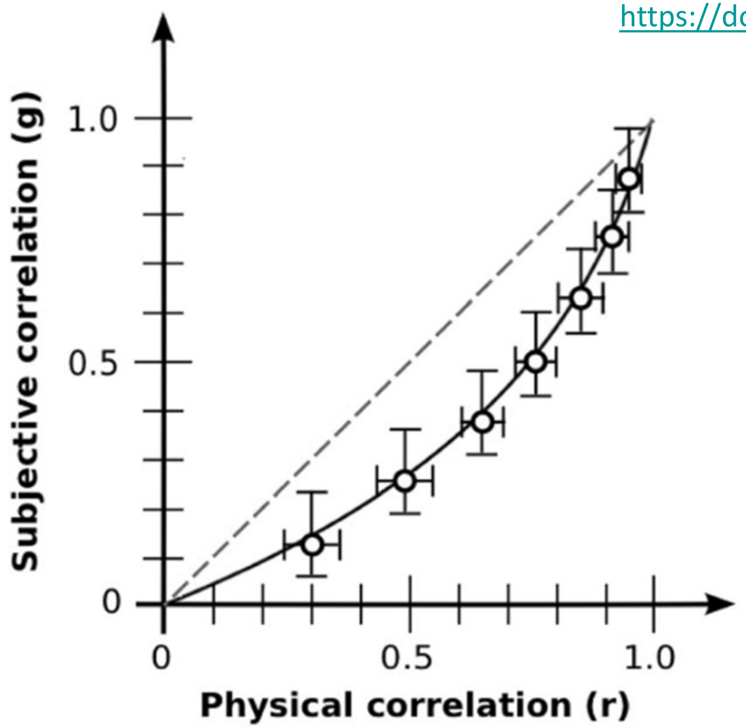
Which of these graphics shows a stronger correlation?

In fact, they both indicate a linear correlation with r = 0.8

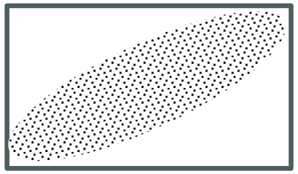
Cleveland et al. asked participants to judge the strength of association between two variables, while varying separately the values of r and the tightness of the scale (from 1 to 4; left diagram = level 2, right diagram = level 4)

Replication : Subjective strength of association is a non-linear function of r^2

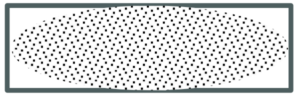
Rensink, R. A. (2017). The nature of correlation perception in scatterplots. *Psychonomic Bulletin & Review*, *24*(3), 776-797.
https://doi.org/10.3758/s13423-016-1174-7



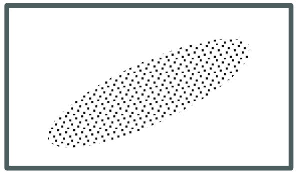
Small surface = high perceived correlation



Big surface = low perceived correlation



Even higher perceived correlation



Rensink (2017) replicates this work while performing very systematic psychophysical experiments on the capacity to estimate and to discriminate different amounts of correlation.

On this basis, he suggests that

- subjects estimate the underlying abstract probability density function (PDF) of the center of the dots (regardless of their shape, size or color)
- Subjects then judge this PDF as an object: they evaluate its size, and particularly its relative width.

Cleveland notices that the non-linear curve varies as $1 - \sqrt{1 - r^2}$.

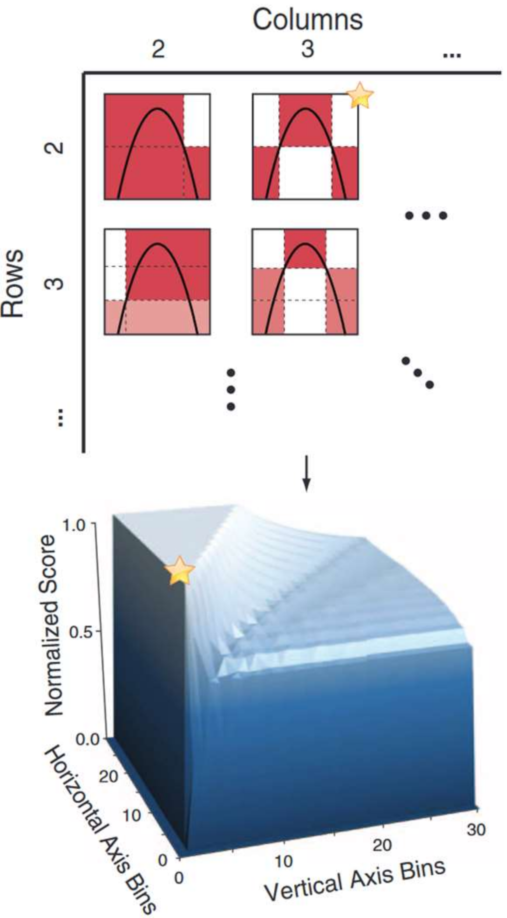
This is inversely related to the surface of the ellipsoid enclosing the dots, relative to the rectangle that frames it, which is $\frac{\pi}{4}\sqrt{1 - r^2}$.

So both the non-linearity and the bias for compactness can be jointly explained by a **recycling of shape analysis**, applied to **the shape of the data**.

This strategy may also allow subjects to detect **any pattern in the data**: coincidences, non-linear curves…

Perhaps the human eye performs something similar to mutual information?

Reshef, D. N., Reshef, Y. A., Finucane, H. K., Grossman, S. R., McVean, G., Turnbaugh, P. J., Lander, E. S., Mitzenmacher, M., & Sabeti, P. C. (2011). Detecting Novel Associations in Large Data Sets. *Science*, *334*(6062), 1518-1524. https://doi.org/10.1126/science.1205438



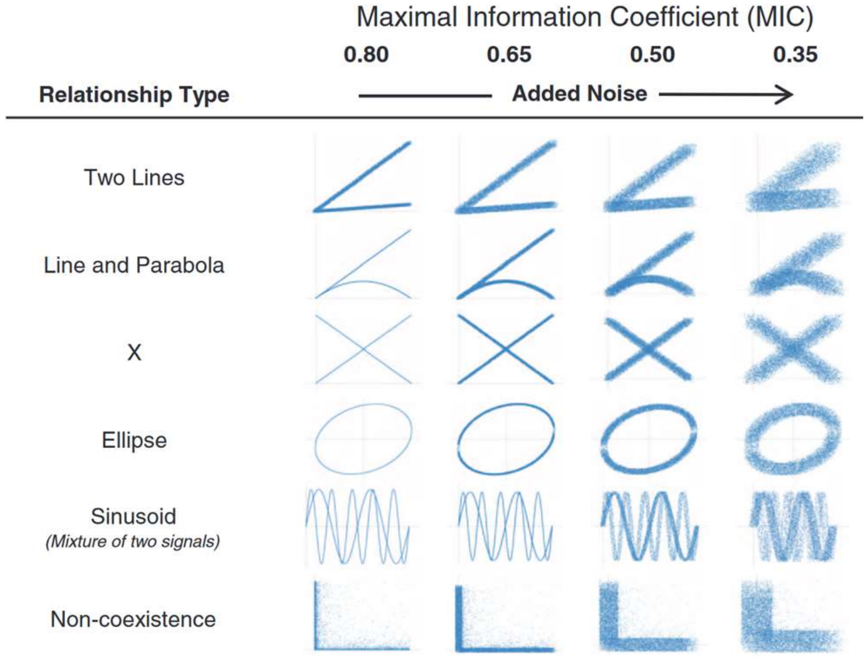
Reshef et al. ask for a measure of correlation that would be able to identify *any* relationship between x and y, linear or not.

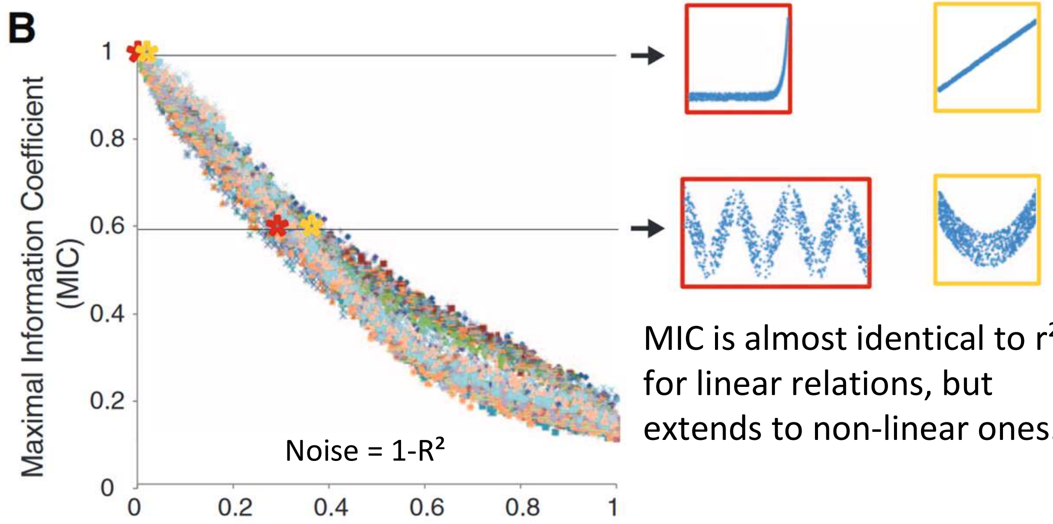
Furthermore, the measure has to be *equitable,* i.e. give the same value to graphics with the same amount of added noise.

They propose such a measure, the MIC (maximal information coefficient), which is based on computing the *mutual information* between x and y over various n x m grids, and choosing the maximum (divided by log min(n,m)).

Like the principal axis, the MIC is symmetrical over x and y.

It would be interesting to examine more closely if the MIC predicts human intuitions of correlation.





MIC is almost identical to r^2 for linear relations, but extends to non-linear ones.

The neural bases of graphicacy

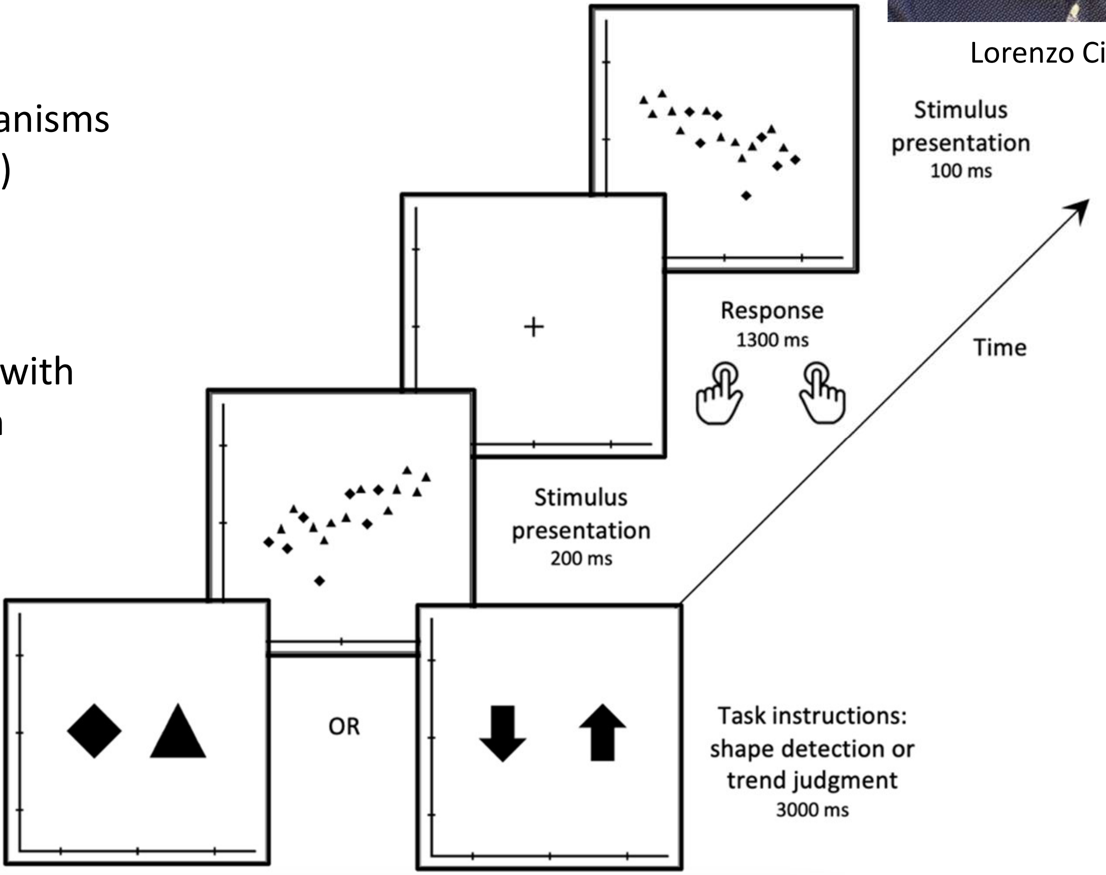
Ciccione, L., & Dehaene, S. (2024, submitted). The neural bases of graph perception: a novel instance of cultural recycling

Lorenzo Ciccione

Prediction 2 of the neuronal recycling hypothesis : The perception of the central tendency in graphics should **recycle** brain areas involved in the perception of objects and their orientation

Nothing is currently known about the brain mechanisms of graphicacy ! (compare to literacy or numeracy…)
We aimed to

1. Identify the main brain areas involved in trend judgments
2. Test the hypothesis that these regions overlap with those involved in computing object orientation
3. Also test their relations to those involved in mathematical cognition
4. Evaluate whether their activation is driven by the t value which determines the strength of the trend and has been shown to drive behavior.



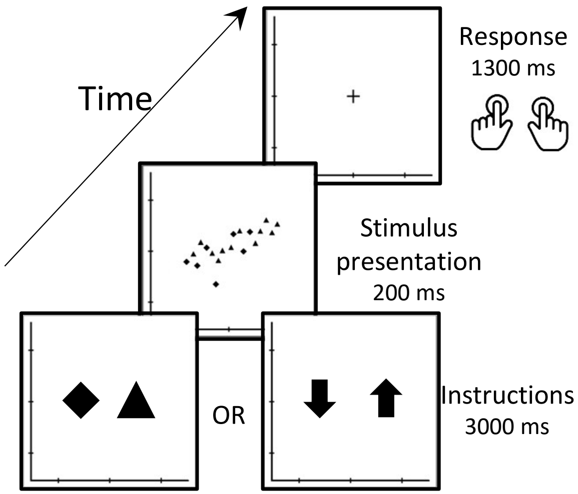
The neural bases of graphicacy : fMRI design

Ciccione, L., & Dehaene, S. (2024, submitted). The neural bases of graph perception: a novel instance of cultural recycling

Graphics (runs 1 & 3)

Goal: for a constant stimulus (graph), determine which areas are active when attending to its global trend or to its local shapes)

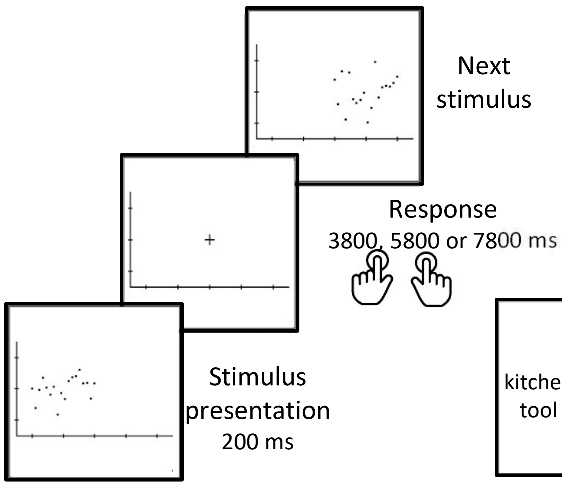
Main task : trend judgment
Control : shape estimation



Graphics (runs 2 & 4)

Goal: Vary the graph parameters (slope, noise, number of points), measure the activation on every trial (with a slow event-related design) and examine its modulation by the t value.

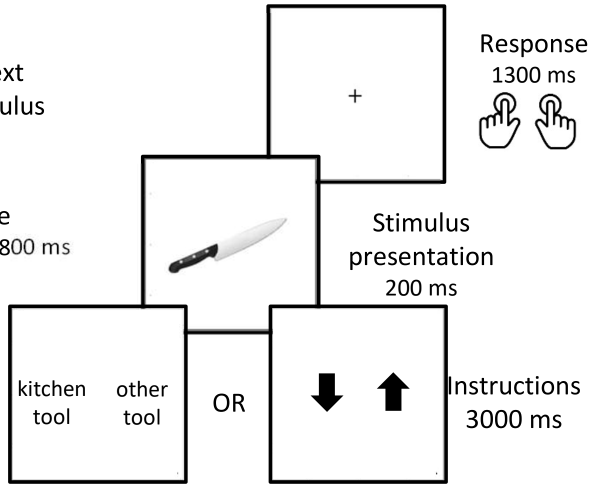
Task: trend judgment



Objects (runs 5 & 6)

Goal: For a constant stimulus (6 objects with 6 possible orientations), determine which areas are active for orientation rather than for category judgment).
Do so *after* the graphics (avoid biasing the subjects to think of graphics as objects)

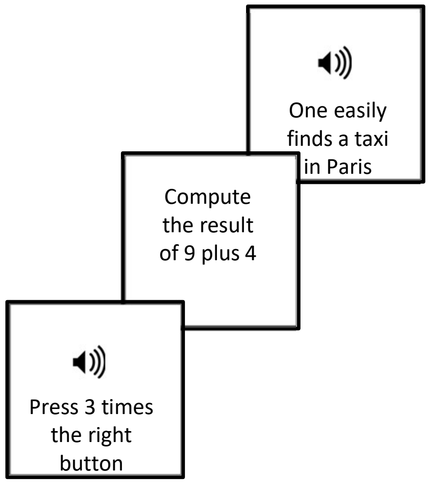
Main task : orientation judgment
Control : category judgment



Math/Language (run 7)

Goal: Localize, in the same subjects, the areas involved in mental arithmetic

Tasks: motor tasks, mental calculations, listening, reading

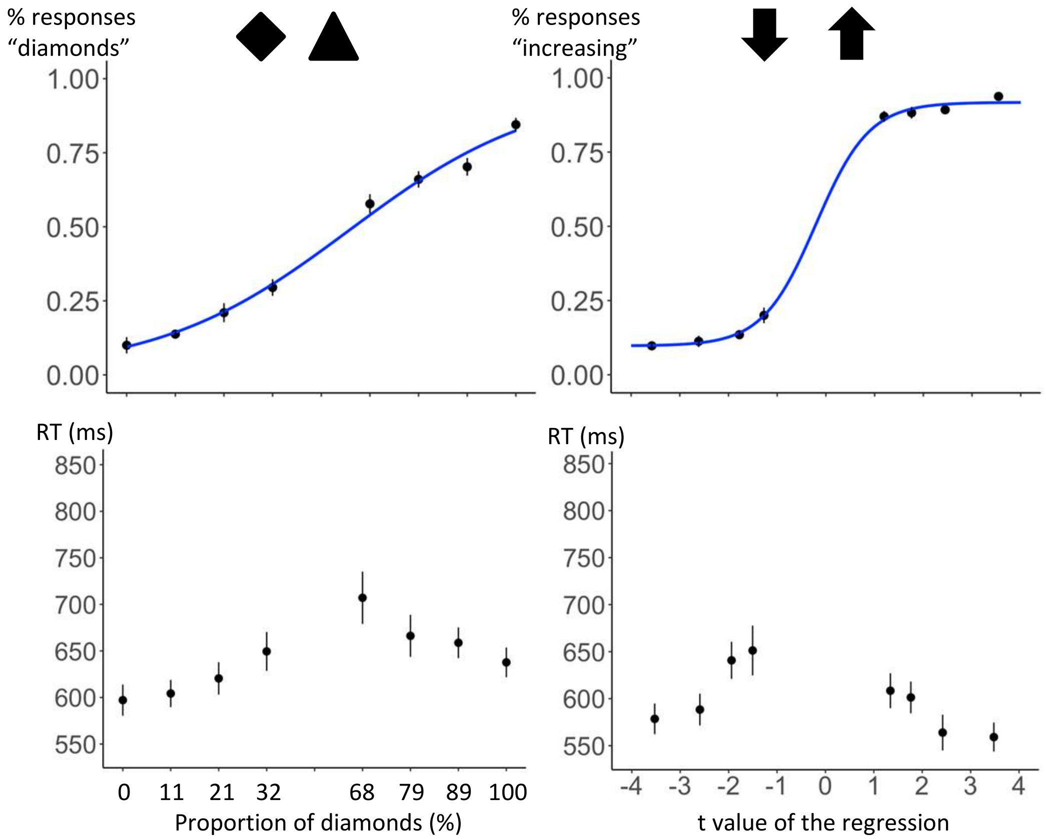


The neural bases of graphicacy: behavioral results during fMRI

Ciccione, L., & Dehaene, S. (2024, submitted). The neural bases of graph perception: a novel instance of cultural recycling

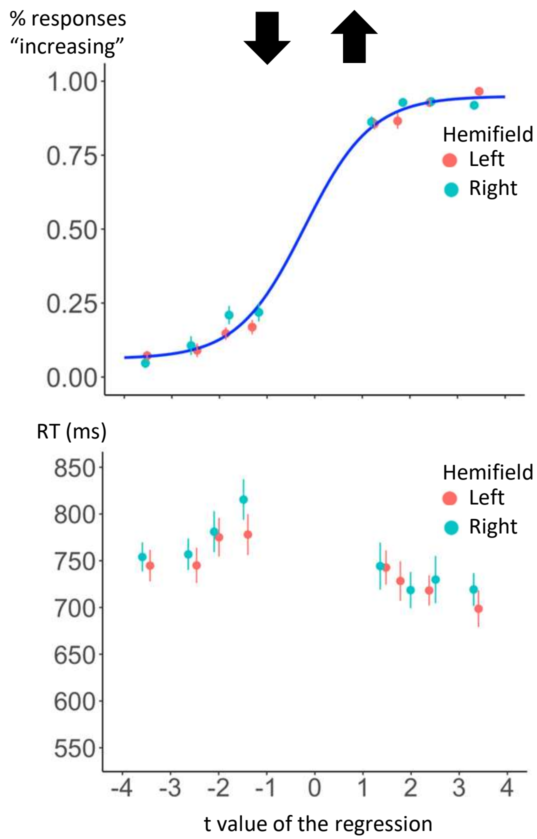
The shape estimation and trend judgement tasks were well-matched, although the shape task was more difficult (over-control for task difficulty)

Shape estimation RUNS 1 & 3 | **Trend judgment** RUNS 1 & 3



The effect of the t value on trend judgment was replicated. No effect of hemifield was found (abstract judgment).

Trend judgment RUNS 2 & 4

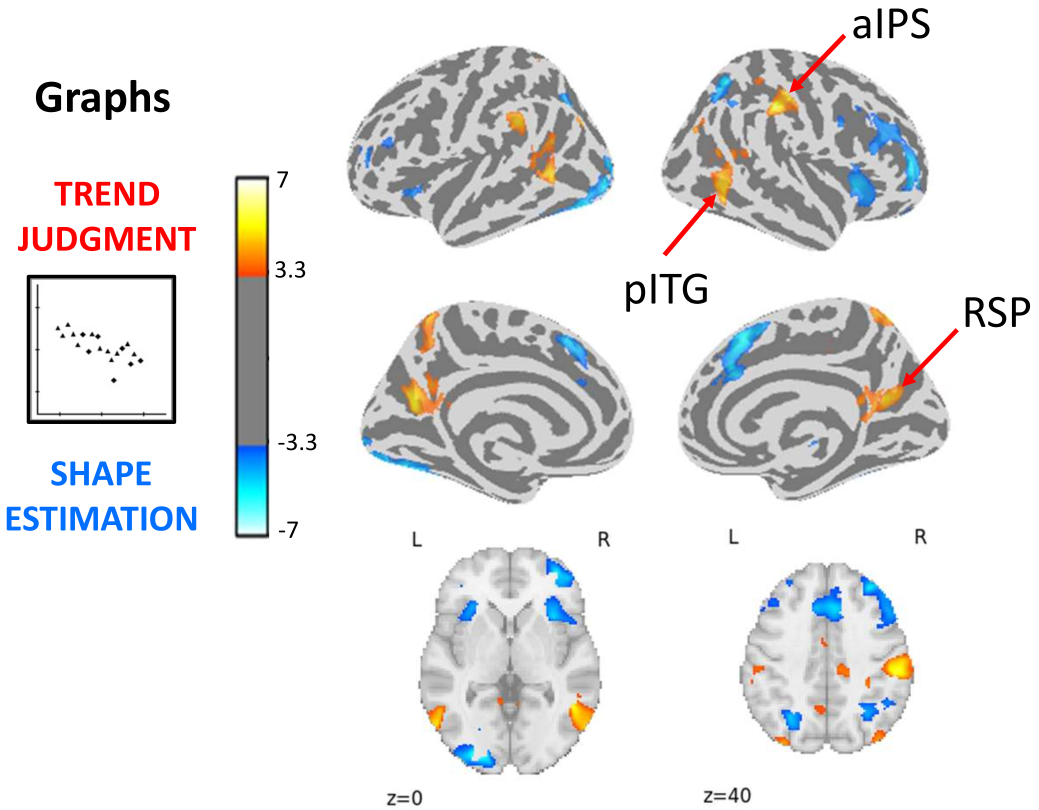


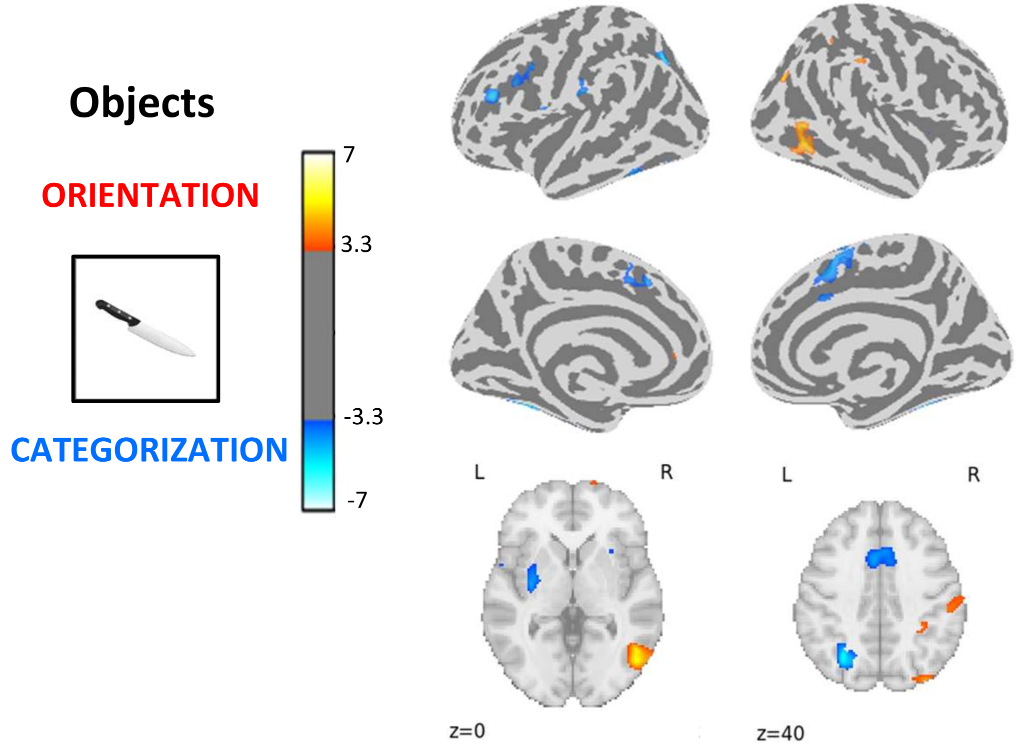
The neural bases of graphicacy

Ciccione, L., & Dehaene, S. (2024, submitted). The neural bases of graph perception: a novel instance of cultural recycling

The trend judgment of scatterplots primarily recruited the right pITG (close to LOC), right aIPS and retrosplenial cortex.

The right pITG/LOC was clearly shared with object orientation judgments in our subjects.





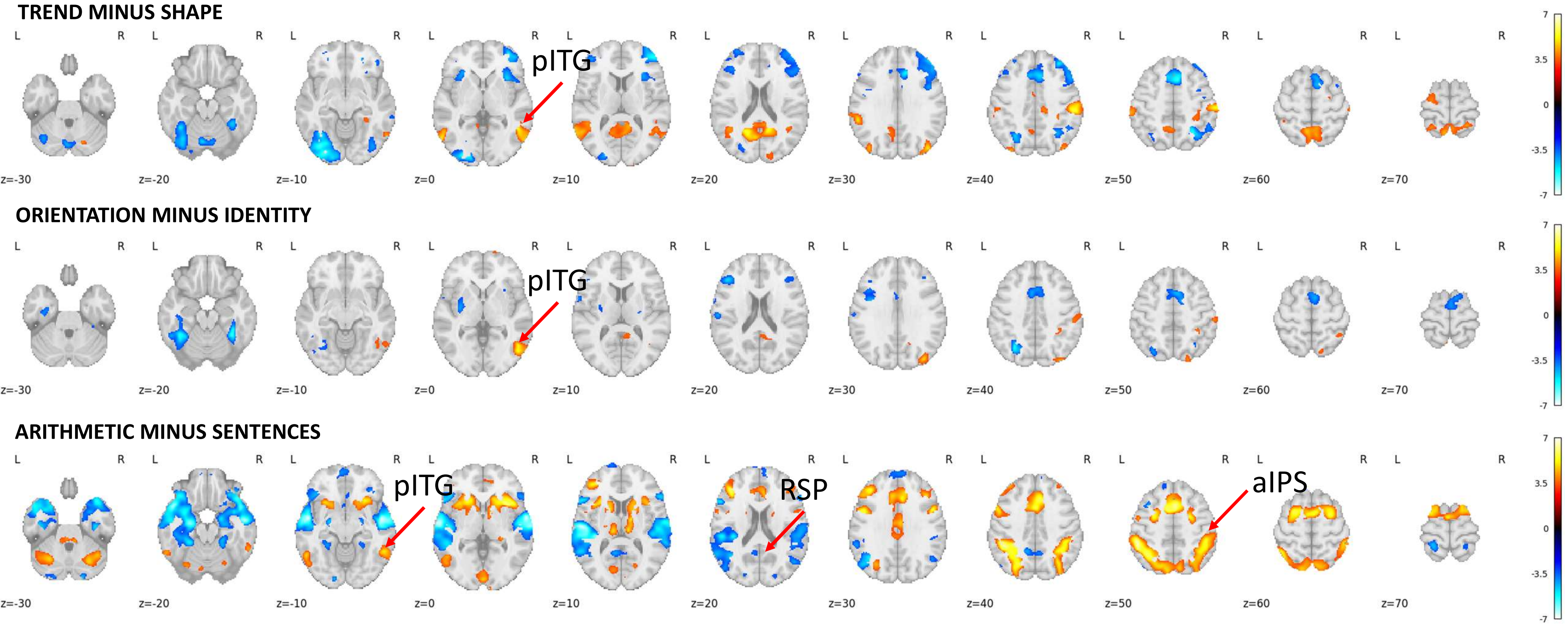
aIPS is active in various numerical, spatial, and mathematical tasks
RSP is an area strongly involved in the representation of space.

The neural bases of graphicacy

Ciccione, L., & Dehaene, S. (2024, submitted). The neural bases of graph perception: a novel instance of cultural recycling

The overlap between trend judgment and object orientation in right pITG/LOC is rather precise.

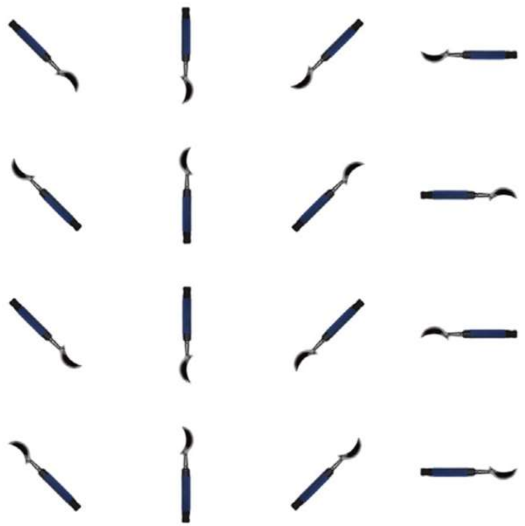
Mental arithmetic activates a similar area in the aIPS but a more anterior pITG region than for graphics and the retrosplenial activation is absent.

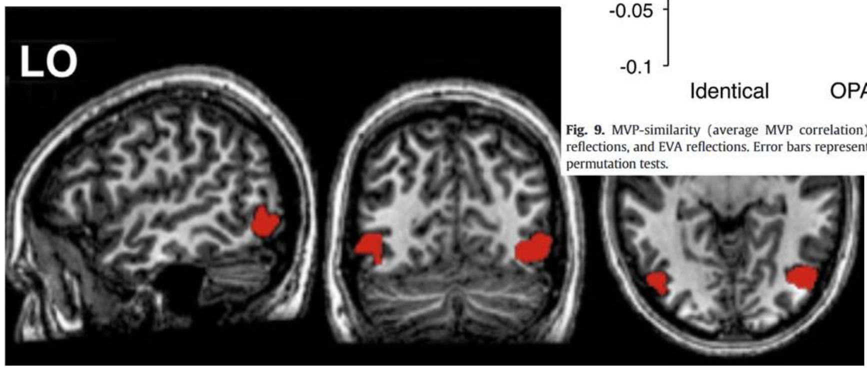


Comparison with other work on orientation perception

Hatfield, M., McCloskey, M., & Park, S. (2016). Neural representation of object orientation : A dissociation between MVPA and Repetition Suppression. *NeuroImage*, *139*, 136-148.

Stimulus set





In experiment 2, Hatfield et al. measured the activation evoked by an object with 8 possible orientations and 2 possible reflections, in subject-specific voxels corresponding to area LO (objects – scrambled).

Multivariate pattern similarity (the resemblance between activation patterns) was strong when the object was just flipped along its principal axis (OPA) but not around the vertical axis (EVA).

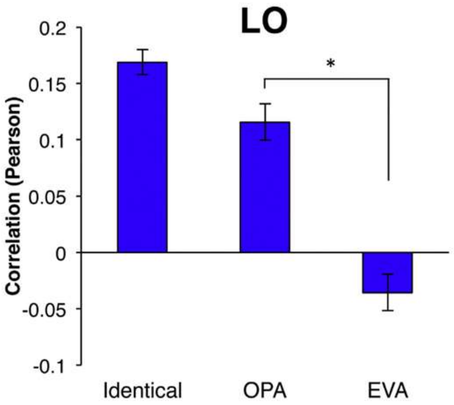
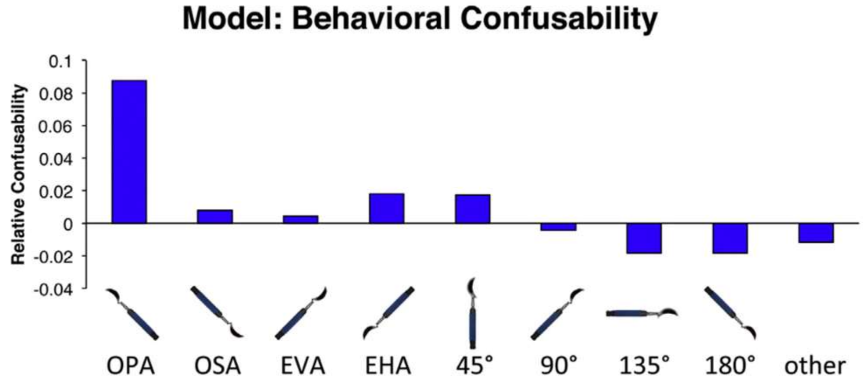
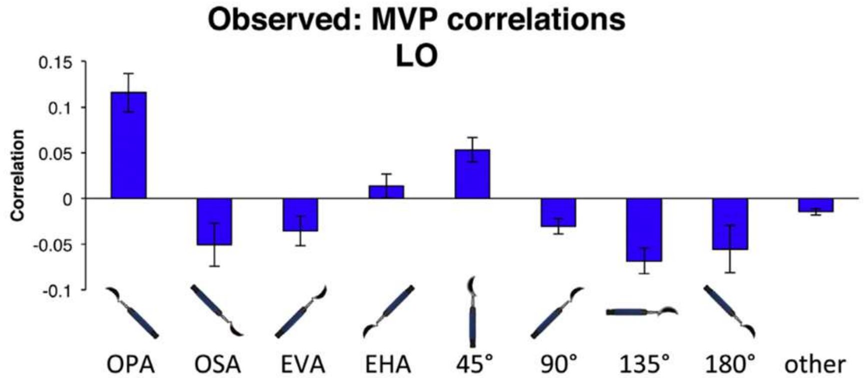


Fig. 9. MVP-similarity (average MVP correlation) for Identical orientations, OPA reflections, and EVA reflections. Error bars represent standard error of the mean from permutation tests.

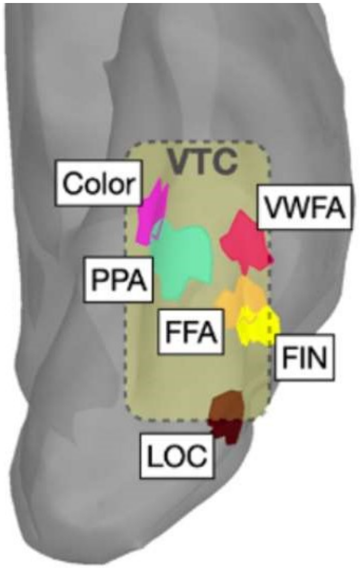
Reference Orientation

Pattern resemblance directly reflected behavioral confusability, and both depended on angle.

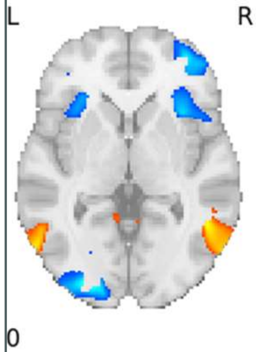




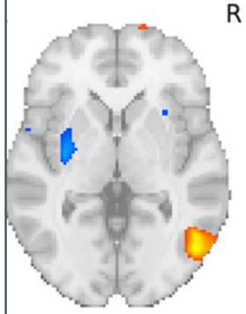
Fusiform imagery node (FIN): Spagna et al, with Liu and Bartolomeo (2024): [-46, -58, -14]



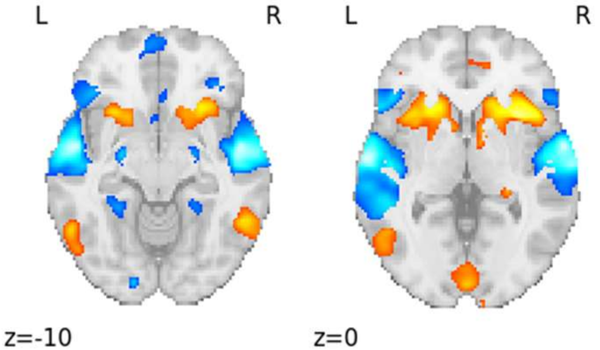
SHAPE



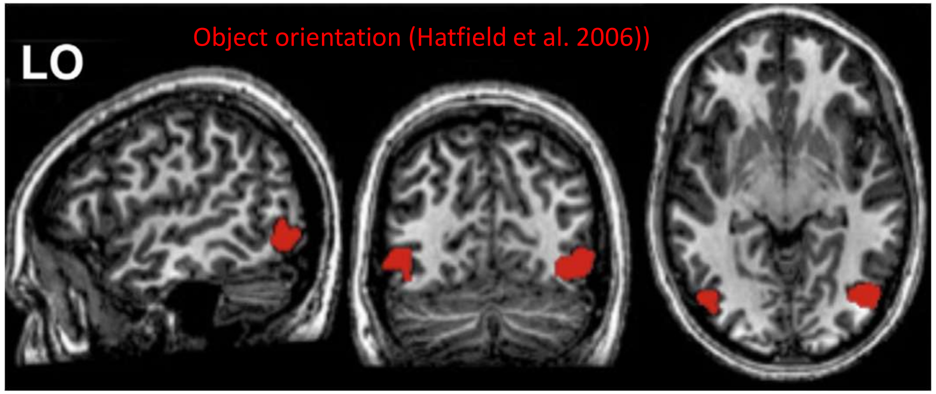
S IDENTITY



ARITHMETIC MINUS SENTENCES

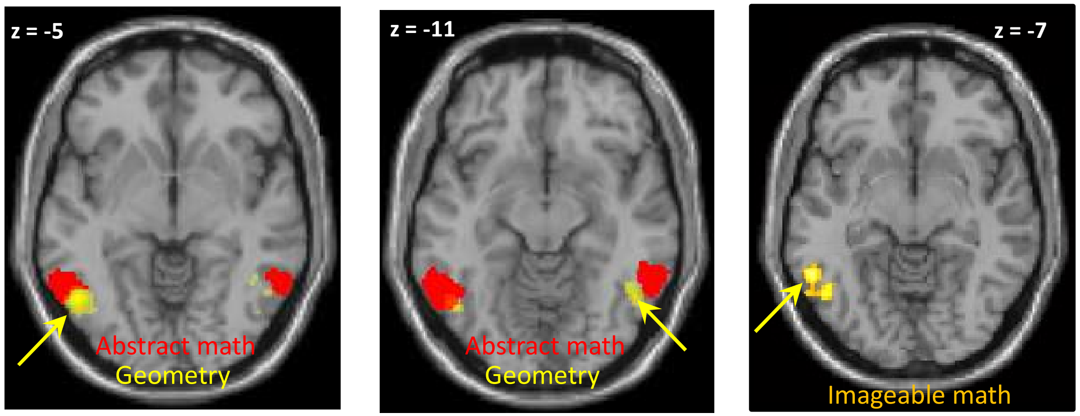


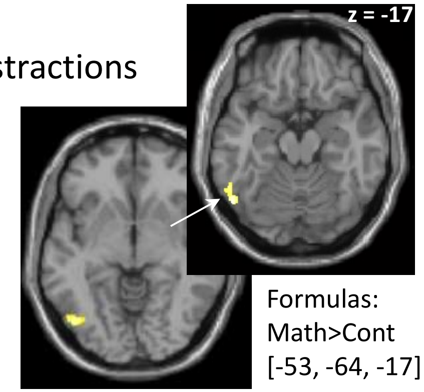
A role for the LOC / pITG / FIN in mathematicians



Our activation during orientation judgement fits nicely with Hatflied et al (2006, left).
However the activation during graph judgements seems slightly anterior, and the activation during arithmetic is clearly more anterior. It may correspond with the FIN.

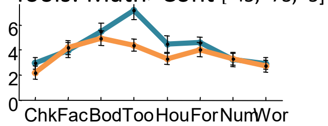
In past work, we also found an activation in pITG, anterior to classical LOC coordinates (red below), when mathematicians answered yes/no questions about complex math facts.
However… a more posterior site was seen for geometric facts (yellow), and also for imageable math.
We also larger activations for formulas and numbers in mathematicians than in controls.
And also a larger activation for tools (?!), exactly at LOC coordinates!
Conclusions: (1) several areas in this region, possibly with a hierarchy of abstractions
(2) in the future, it will be essential to look at this in single subjects at 7T.





Formulas: Math>Cont [-53, -64, -17]

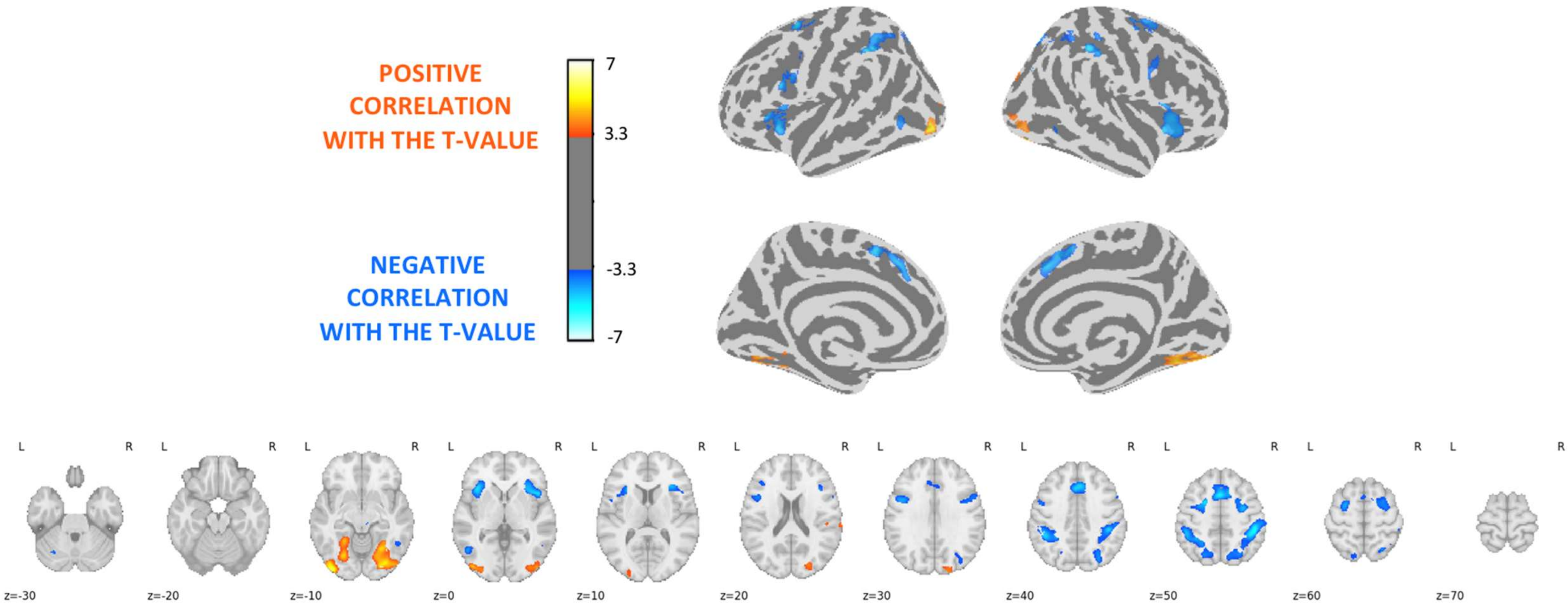
Tools: Math>Cont [-45, -73, -5]



The neural bases of graphicacy: effect of task difficulty

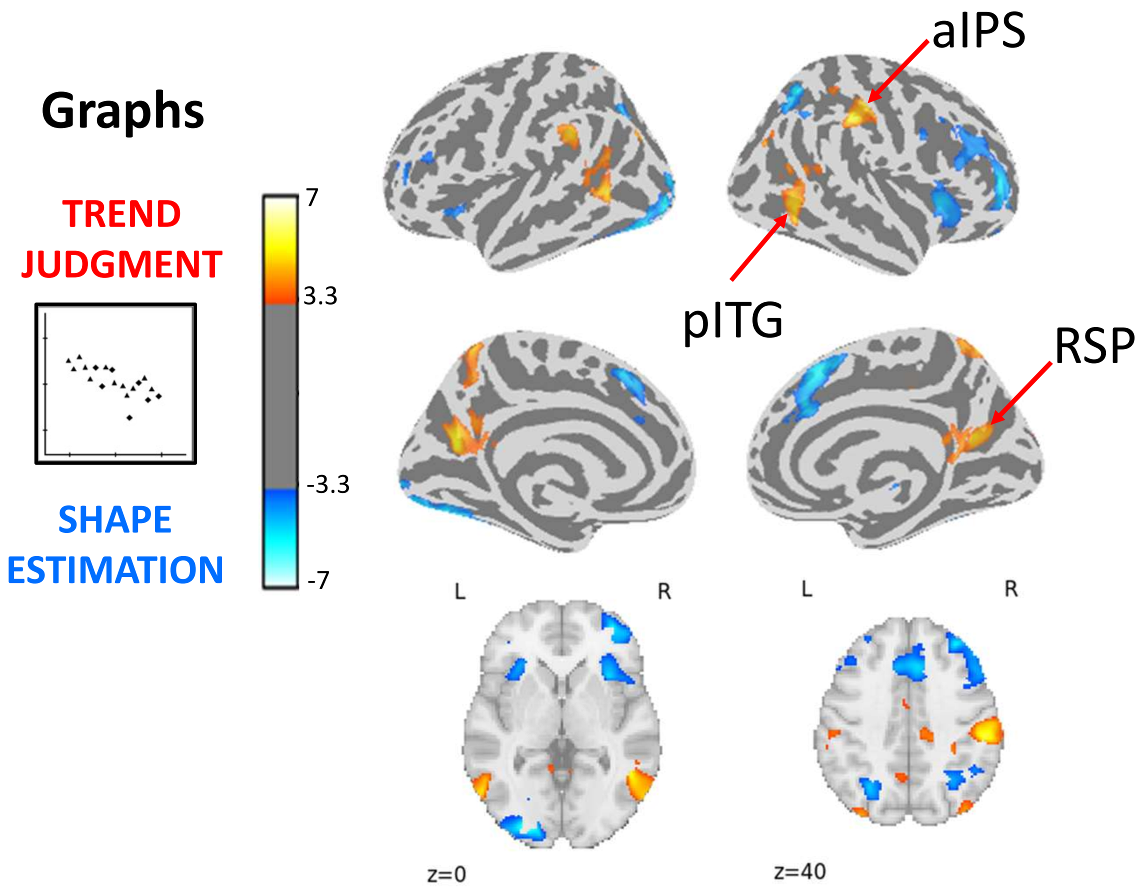
Ciccione, L., & Dehaene, S. (2024, submitted). The neural bases of graph perception: a novel instance of cultural recycling

Activation in aIPS, PFC and anterior insula was strongly modulated by the difficulty of the trend judgment task (blue areas).
This difficulty effect was well captured by the t value, as in behavior (even when an additional regressor for noise level was present)



The neural bases of graphicacy

Ciccione, L., & Dehaene, S. (2024, submitted). The neural bases of graph perception: a novel instance of cultural recycling



Tentative conclusions and a putative model:

A right lateralized network is involved in graphic perception.

The **pITG/LOC** may extract the objective data about the graphic, particularly the shape of the data, such as

- the orientation of its principal axis
- but also the width along its perpendicular secondary axis, which is crucial for the determination of the strength of the linear correlation in a graphic (Rensink, 2017).

Multiple ventral temporal areas seem to be involved around LOC and anterior to it, some of which are involved in high-level geometric concepts and are more developed in mathematicians.

Those objective data would be fed to higher areas such as

- IPS, where the actual **extraction of quantitative graphic statistics**, such as its slope, t value, and correlation strength might occur
- RSP for higher-level, perhaps hierarchical synthesis of the graphic components?

Cours 2024-2025:

La perception des graphiques:
un nouvel exemple de recyclage neuronal
The perception of graphics : a new example of neuronal recycling

Stanislas Dehaene
Chaire de Psychologie Cognitive Expérimentale

Cours n°5

La grammaire des courbes et des graphiques

The grammar of curves and graphics

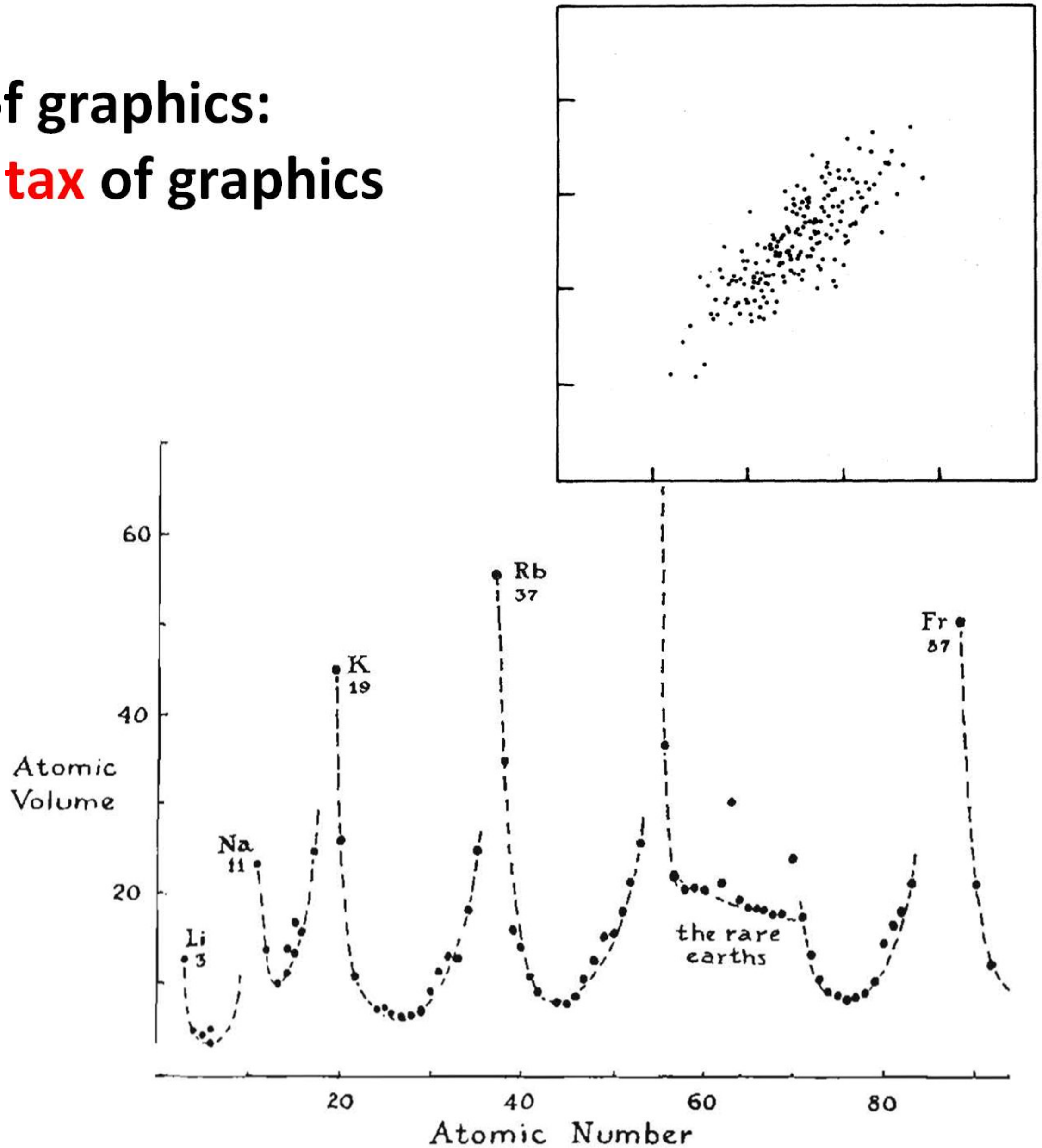
A cognitive requirements of graphics: Grasping the compositional syntax of graphics

In the past courses, we showed that a scatterplot (a broad array of data points) can be grasped in parallel.

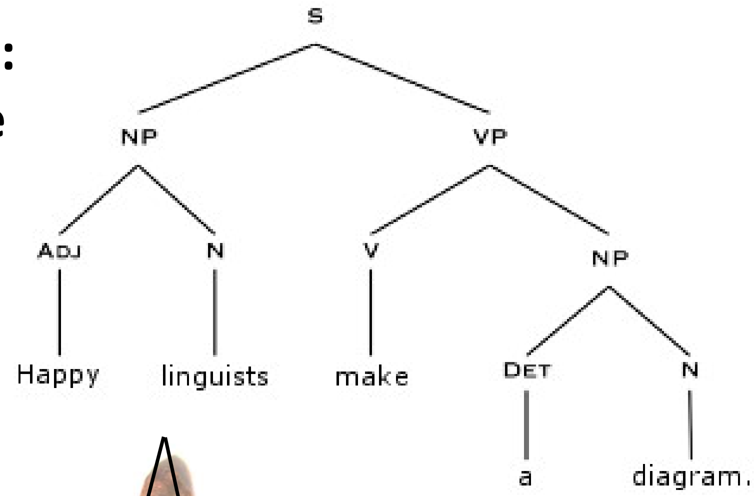
However, the linear trends that we studied in the lab are extremely impoverished compared to real graphics.

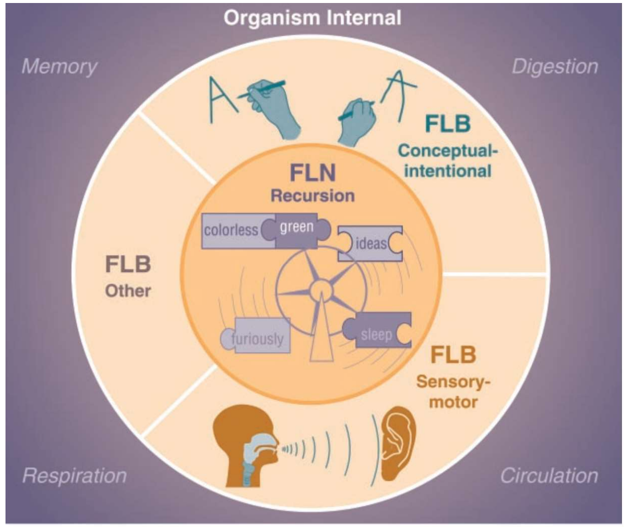
Real graphics involve a much more complex and hierarchical perceptual scene.

- Some points may be special (outliers)
- The curves may be composite and have a syntax of their own
  → A language of functions
- The whole scene must be parsed
  → A language of graphics



A recursive language of geometry : Summary of my 2023-2024 course

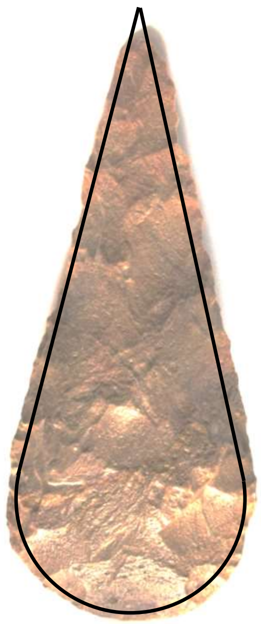




Humans are characterized by their capacity to **form nested, recursive structures** – and this ability is s**pecifically human**.

It allows children to create new concepts by **recombining existing ones into novel expressions** (e.g. quadrilateral = four-sided figure; square = quadrilateral with equal sides)

«A square of circles »



« 2 lines of equal length touching a circle »

- Hauser, Chomsky and Fitch (2002): **Recursive merge** lies at the core of the **language** faculty.
- Tecumeh Fitch's dendrophilia hypothesis: Recursive "tree" structures are omnipresent in human cognition: **language, music, math, science, tools**…
- Hauser and Watumull's **Universal Generative Faculty**.

Fitch, W. T., Hauser, M. D., & Chomsky, N. (2005). The evolution of the language faculty : Clarifications and implications. *Cognition*, *97*(2), 179-210; discussion 211-25.
Hauser, M. D., Chomsky, N., & Fitch, W. T. (2002). The faculty of language : What is it, who has it, and how did it evolve? *Science*, *298*(5598), 1569-1579.
Hauser, M. D., & Watumull, J. (2017). The Universal Generative Faculty : The source of our expressive power in language, mathematics, morality, and music. Journal of Neurolinguistics. https://doi.org/10.1016/j.jneuroling.2016.10.005

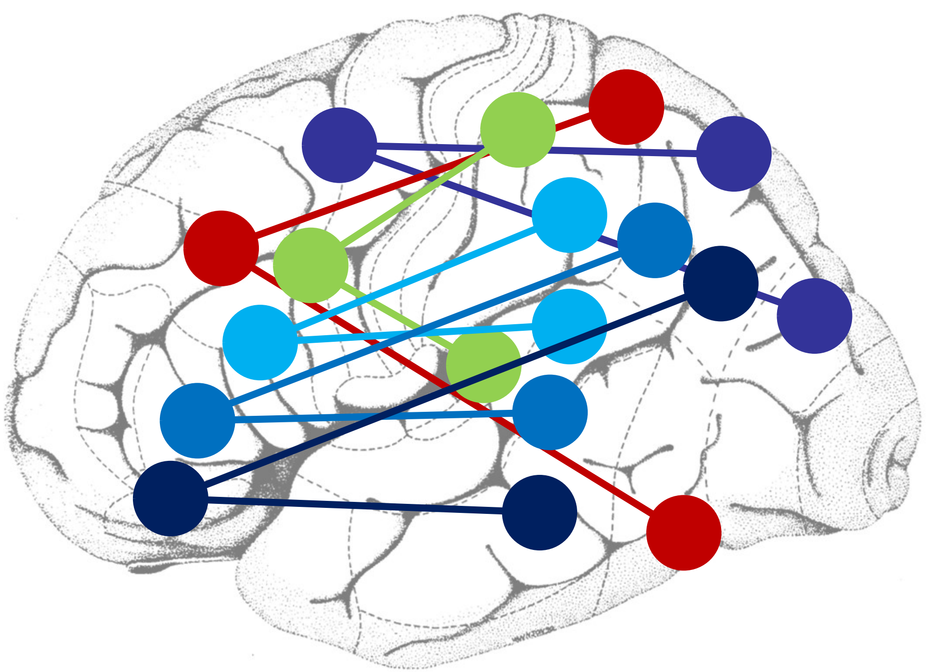
Multiple languages of the brain

Dehaene, Al Roumi, Lakretz, Planton and Sablé-Meyer, Symbols and mental programs: a hypothesis about human singularity. *TICS*, 2022

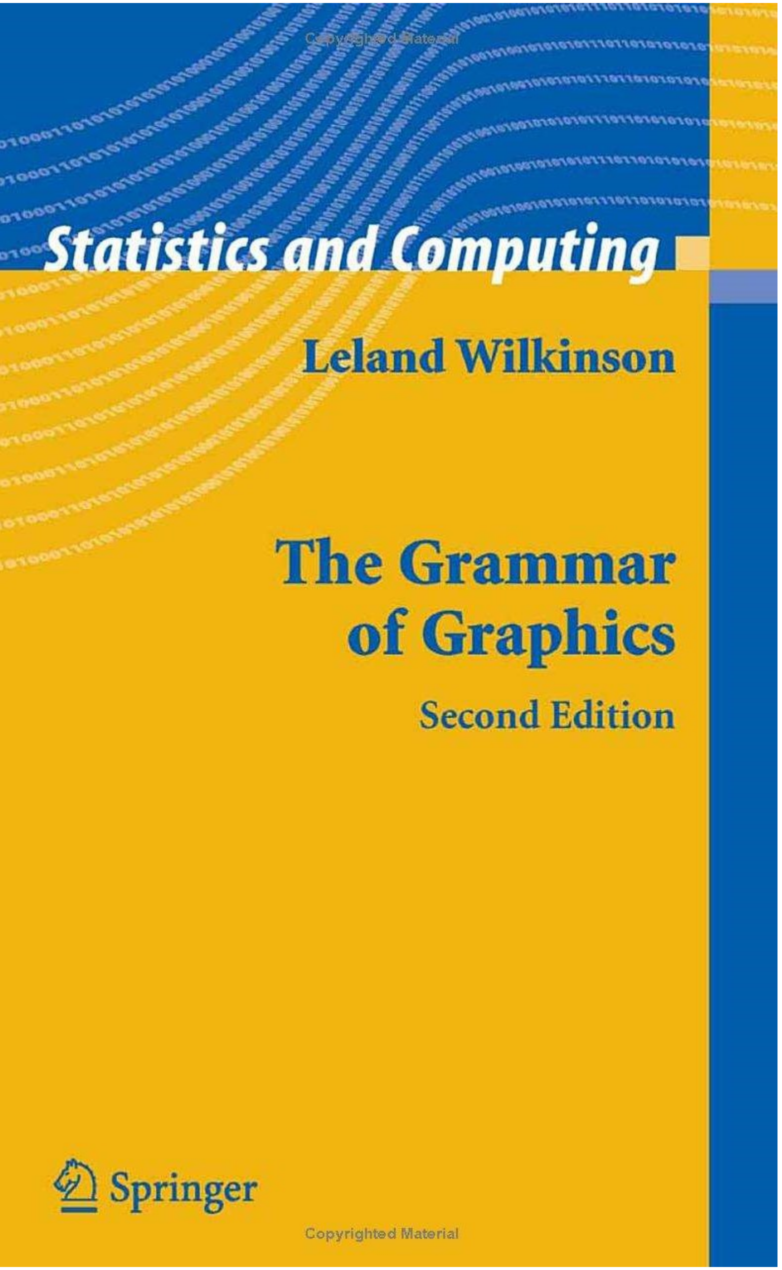
In humans, **several parallel networks**, involving different sectors of prefrontal cortex, may have evolved a capacity for recursive composition.

Each of those "languages of thought"

- **discretizes concepts**
- assigns them **symbols that compose recursively**



Shared principles	**Programing style**	**Domain-specific primitives**
Discrete **symbols**	**Symmetrical structures:**	**Mathematics:** number, set, distance, space…
Composition by concatenation, iteration and recursion	Repetition with variation	**Spatial sequences:** location, distance, rotation, symmetry…
Formal grammar involved in both comprehension and production	Nested loops for i=1:n	**Music:** pitch, chord, rhythm, number…
Compression by searching for the **minimal description length** (MDL)	**Linguistic structures:**	**Phonology:** vowel, consonant, phonetic features…
	Labeled trees created by Merge	**Syntax:** parts of speech, syntactic features…
	Avoid repetition (antisymmetry)	**Semantics:** object, time, aspect, mental verbs…



The grammar of graphics (2005)

« few have viewed quantitative graphics as an area that has peculiar rules and deep grammatical structure. »

Instead of treating visualization as a collection of pre-defined chart types, Wilkinson argues that graphics must be constructed hierarchically according to a language-like representation.

Starting from a structured dataset, the grammar of graphics specifies :

- The **mapping** of variables onto **visual dimensions** (e.g., x-position, y-position, color, size, shape).
- Which **Statistical Transformations** to apply (e.g., counting, averaging, binning).
- Which **Scales** to apply, e.g.
    - A linear or logarithmic scale for a continuous or ordinal variable.
    - A categorical color scale for different groups.
- Which **Geometric Objects** to use
    - Points for a scatter plot.
    - Bars for a bar chart.
    - Lines for a line graph.
- Which **Coordinate System** to use, e.g. Cartesian (standard x-y plane) or Polar (for pie charts or radar plots).
- (optionally) how to “**facet**” the, i.e. split it into subsets displayed in **separate panels** (Tufte’s small multiples)

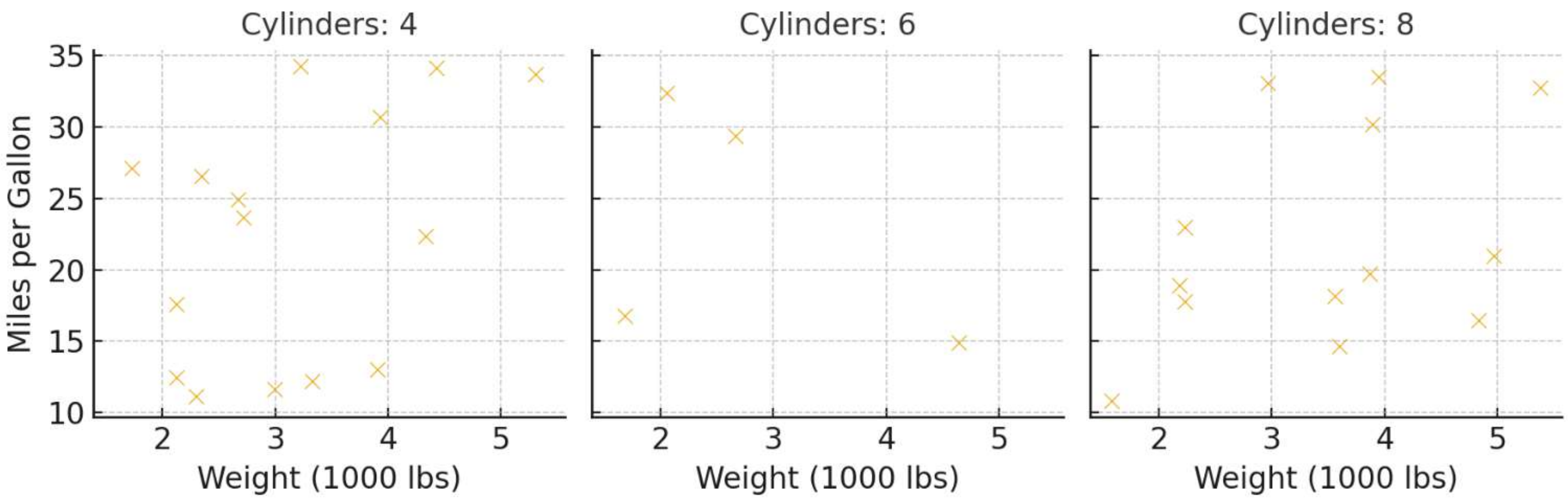
Example of ggplot2 code (directly inspired by « The grammar of graphics »)

```
ggplot(mtcars, aes(x = wt, y = mpg)) +   # Define variables & mappings
  geom_point() +                         # Choose geometry (points)
  scale_x_continuous() +                 # Define scales
  scale_y_continuous() +
  coord_cartesian() +                    # Set coordinate system
  facet_wrap(~cyl)                       # Apply faceting (if needed)
```

My proposal for future research :

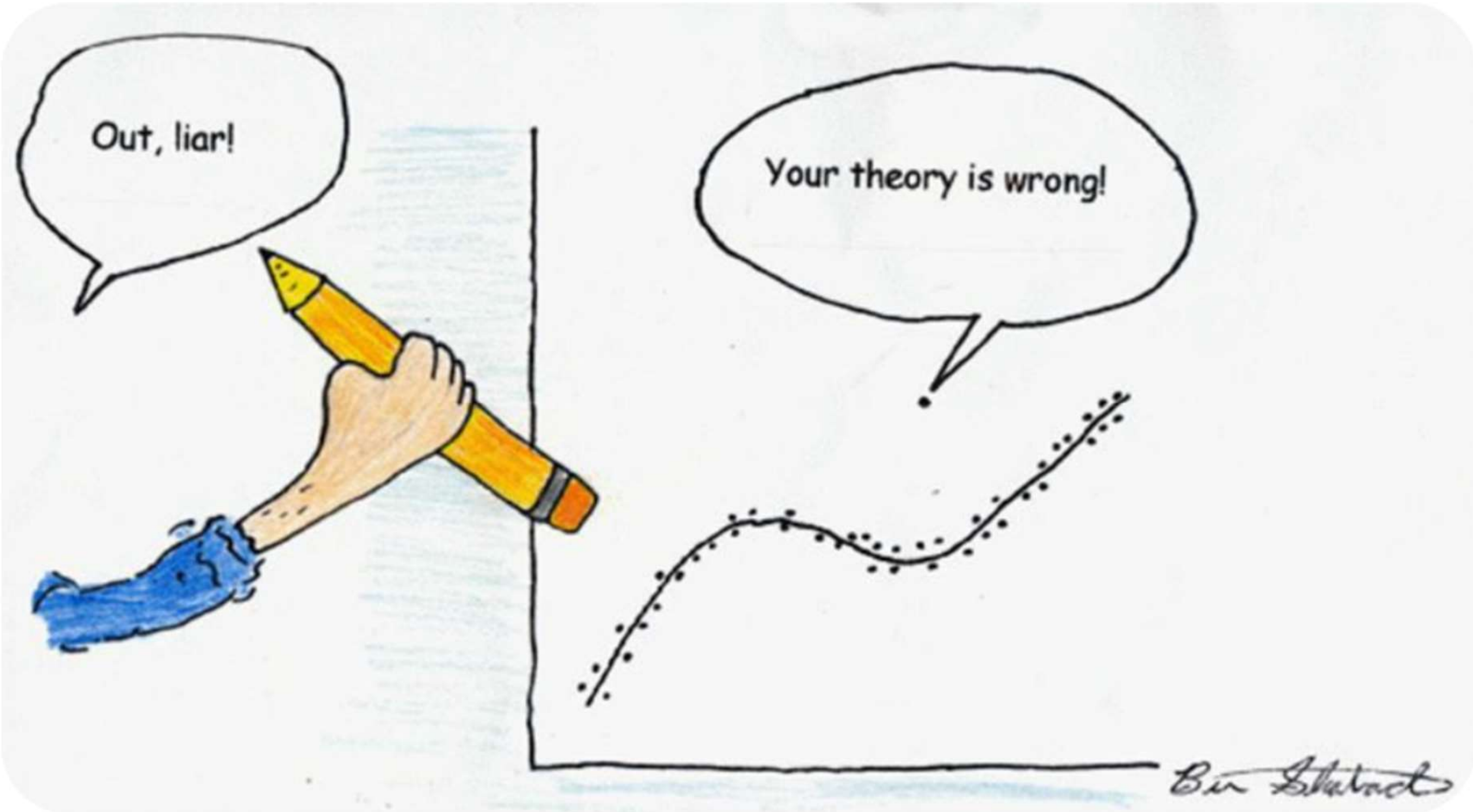
The viewer “undoes” this production process to recover the **grammatical tree of intentions** behind a graphic.

Graphic production and perception are linked.



Question 1. Can humans detect and reject outliers ?

Ciccione, L., Dehaene, G., & Dehaene, S. (2023). Outlier detection and rejection in scatterplots : Do outliers influence intuitive statistical judgments? *Journal of Experimental Psychology: Human Perception and Performance*, *49*(1), 129-144. https://doi.org/10.1037/xhp0001065



Real-life data processing often requires detecting and rejecting rare data points that come from another distribution ("outliers").

To do so, statisticians invented procedures of "robust regression".

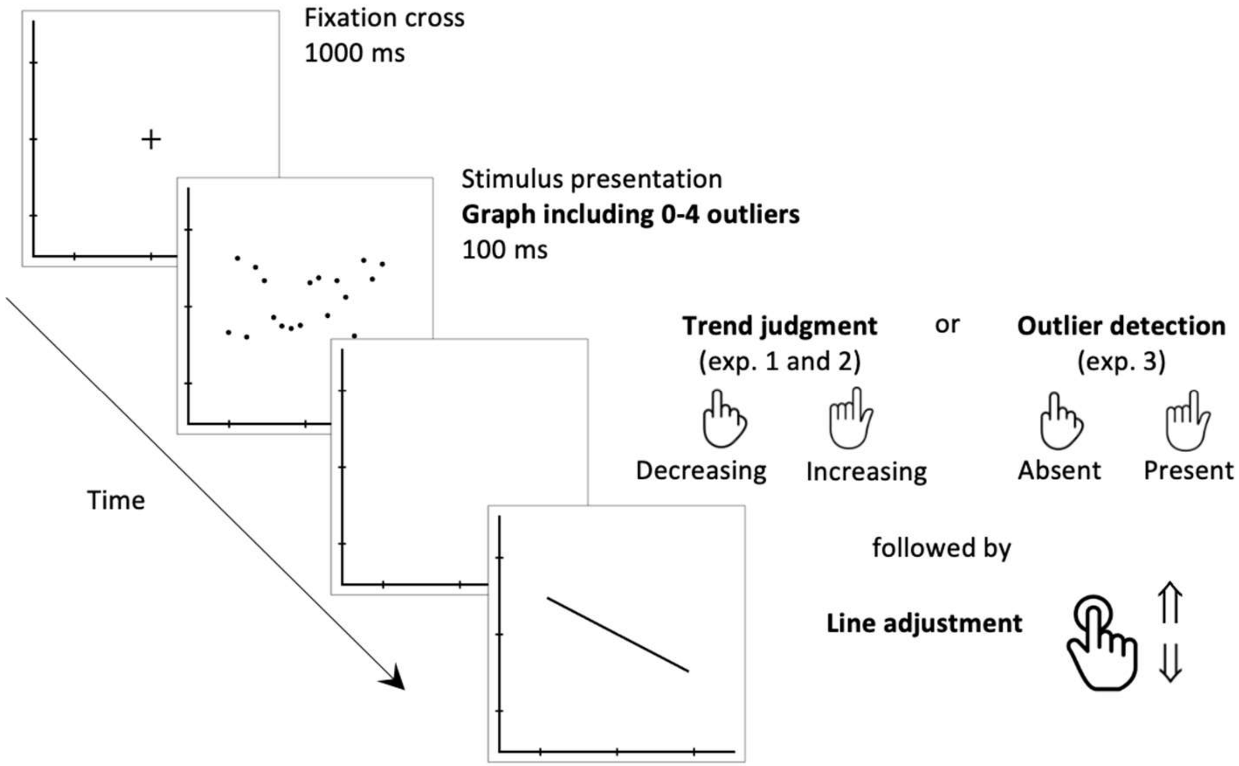
We asked

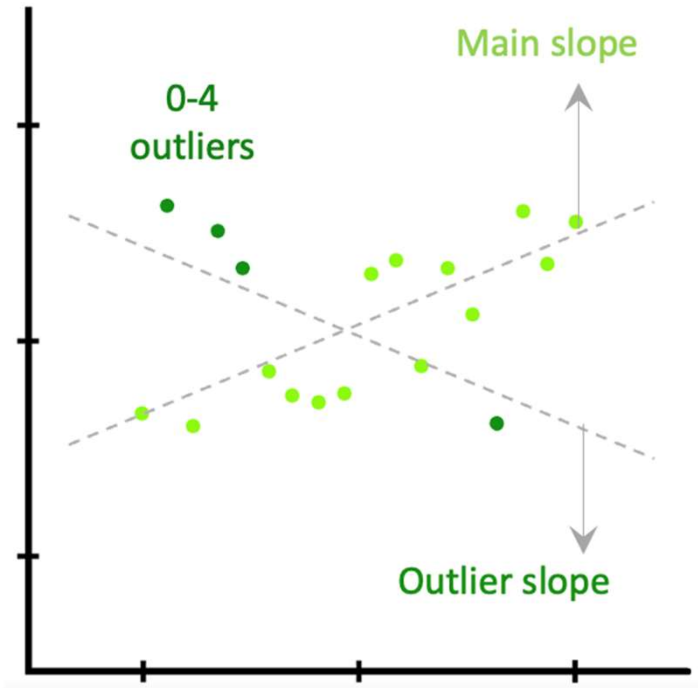
- Can humans perform robust regression? Do they spontaneously reject outliers?
- Does their performance depend on the attention drawn to the possible presence of outliers?
- Does this depend on the number and distance of the outliers?
- What is the mechanism for outlier detection?
- Is detection the same as rejection?

Can humans detect and reject outliers ?

Ciccione, L., Dehaene, G., & Dehaene, S. (2023). Outlier detection and rejection in scatterplots : Do outliers influence intuitive statistical judgments? *Journal of Experimental Psychology: Human Perception and Performance*, *49*(1), 129-144. https://doi.org/10.1037/xhp0001065

We asked for the same two task as before, but now on the same trial : **trend judgment** plus **line adjustment**.
The displays had 18 dots, but **0-4 outlier data points** were generated from another regression line with a different slope.
We varied the **attention to outliers**: In experiment 1, participants were not informed of the presence of outliers. In experiment 2, participants were informed that some outliers could be present, and were asked to try to ignore them. Experiment 3 further emphasized outliers by first asking for explicit outlier detection before the slope adjustment task.





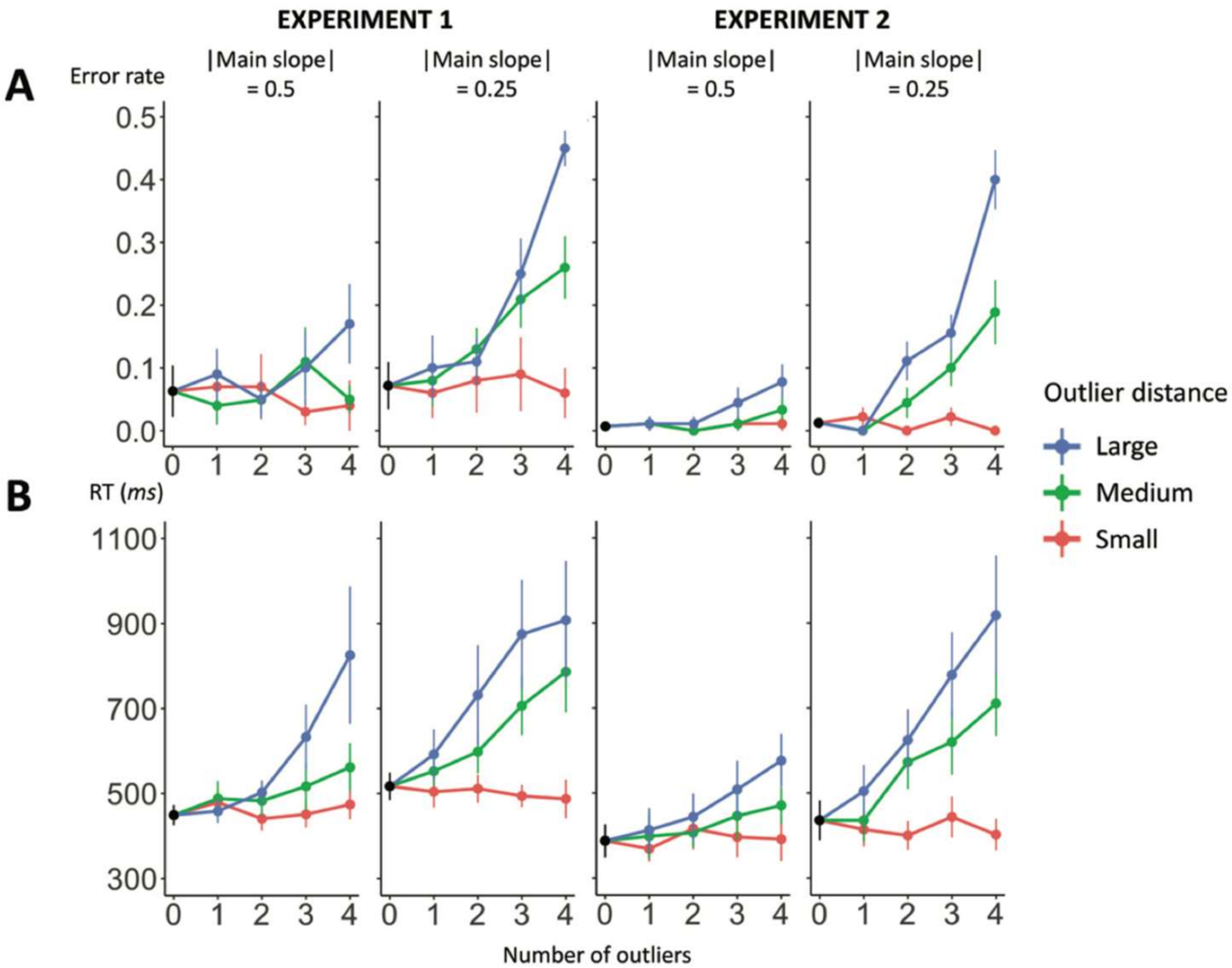
Can humans detect and reject outliers ?

Ciccione, L., Dehaene, G., & Dehaene, S. (2023). Outlier detection and rejection in scatterplots : Do outliers influence intuitive statistical judgments? *Journal of Experimental Psychology: Human Perception and Performance*, *49*(1), 129-144. https://doi.org/10.1037/xhp0001065

In the **trend judgment task**, participants were clearly influenced by the presence and **number of outliers** (on the x axis).

- They made more errors
- And they were slower (and this influence was not just due to a change in the overall slope, which made the task more difficult)

They were more influenced
- When the outliers were more distant from the main regression line
- when the main trend was less obvious (shallower main slope).



Can humans detect and reject outliers ?

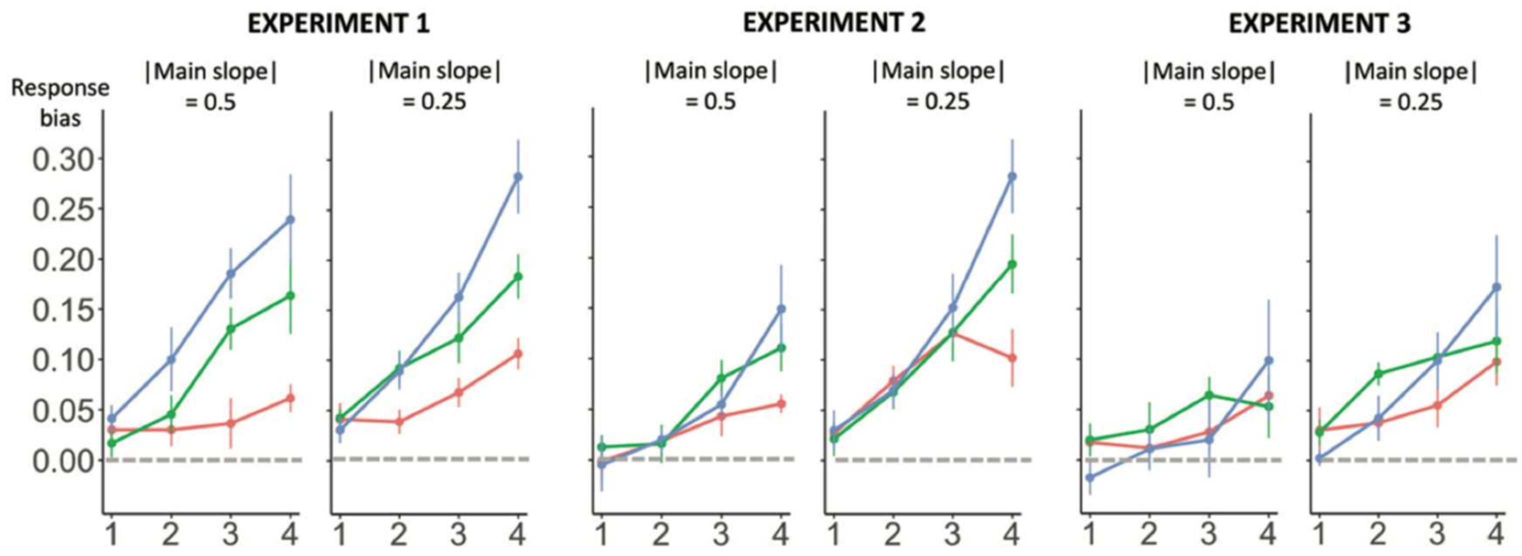
In the **line adjustment task**, participants were also influenced by the presence and number of outliers (x axis).

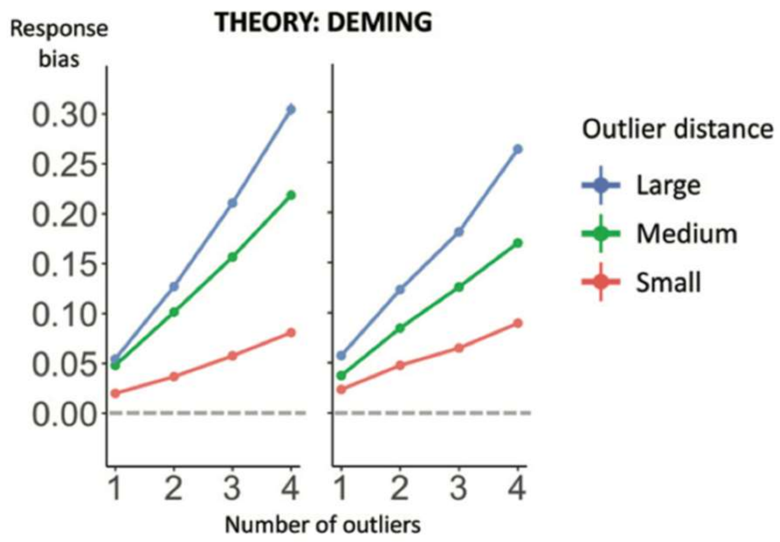
Here we plot the data as a bias, i.e. a change in slope relative to the situation without outliers.

The responses are again affected by the number of outliers, and their distance to the main line.

The results are also strongly affected by the attention to ouliers: their influence decreases from experiment 1 to 2 and 3.

In the absence of knowledge of the outliers (exp. 1), participants incorporate them fully in their judgements, exactly as the Deming regression would predict.





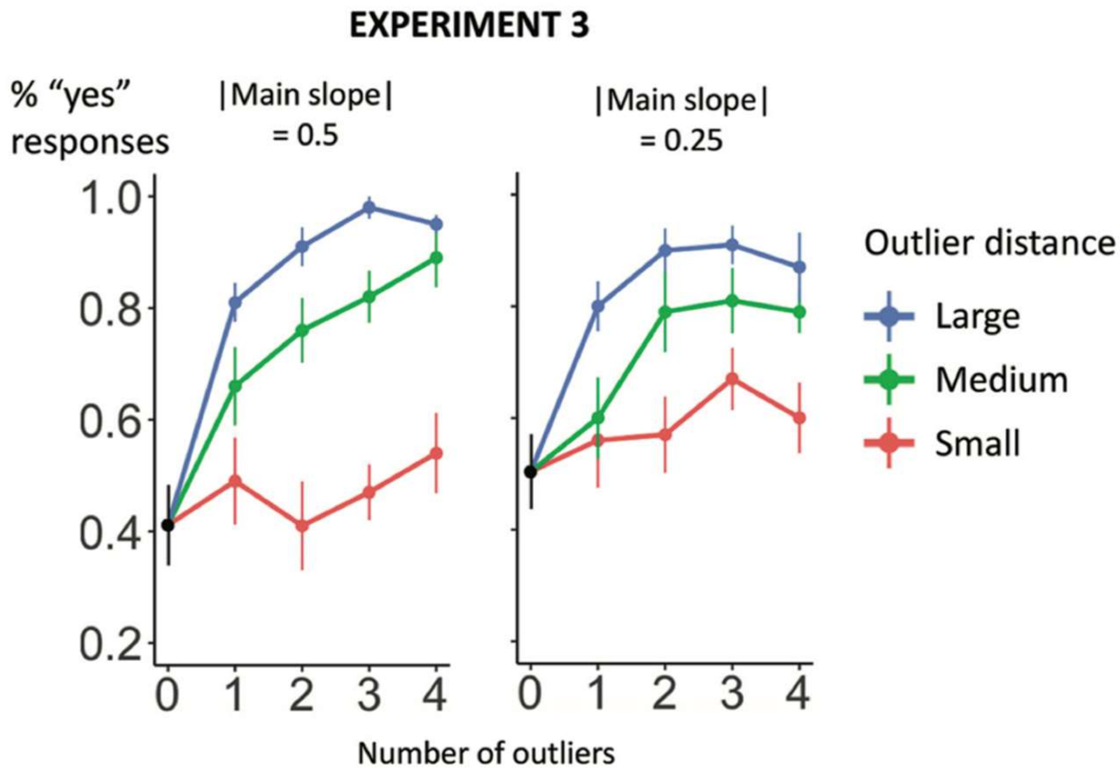
In experiments 2 and 3, where participants are told about outliers, they manage to reduce their influence, particularly when they are easier to detect (because the main slope is large and therefore more obvious).

Can humans detect and reject outliers ?

Ciccione, L., Dehaene, G., & Dehaene, S. (2023). Outlier detection and rejection in scatterplots : Do outliers influence intuitive statistical judgments? *Journal of Experimental Psychology: Human Perception and Performance*, *49*(1), 129-144. https://doi.org/10.1037/xhp0001065

How good were humans in the **explicit detection** of outliers?

They responded that outliers were present even when they were not – but those responses clearly increased with the actual number of outliers, only when they were sufficiently distant from the main regression line;

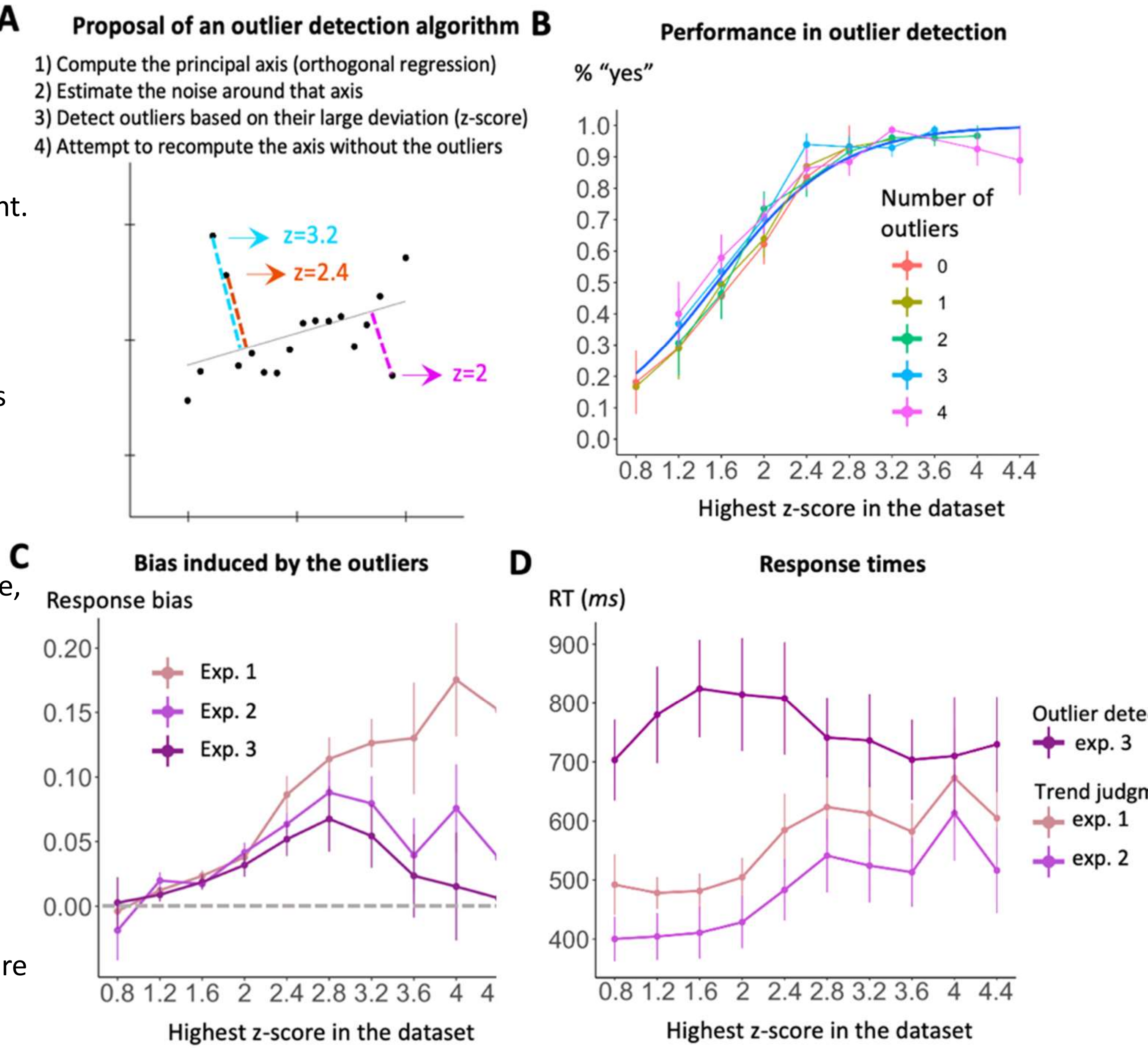


The algorithm for human outlier detection

The results could be explained by an explicit model for outlier detection, based on the z score of each data point. Indeed, the subjects' detection responses were *better* explained by the maximum z score than by the fact that we drew 1, 2, 3 or 4 outliers.
Interestingly, the data was again better modeled by the Deming distance, and also by assuming that participants estimate the standard deviation of the data across multiple trials, not just the current display.
What was the influence of the z score on trend judgment?

- In the range of $z < 2.4$, outliers had a strong influence, regardless of the instructions
- About $z = 2.4$, we began to see a strong influence of attention : the influence of outliers could be attenuated (but not completely eliminated).

Response times confirm that outlier detection is a slow process (slower than trend judgment) whose speed depends on the distance of z to the boundary 2.4.
Conclusion : Fast parallel trend judgment can be supplemented with a slow, serial process of outlier detection – but even when outliers are detected, they are not necessarily rejected!



A related question: what is the “loss function” for graphic perception?

Ryu, H. X., & Srinivasan, M. (2023). What Loss Functions Do Humans Optimize When They Perform Regression and Classification. bioRxiv: The Preprint Server for Biology, 2023.09.19.558376. https://doi.org/10.1101/2023.09.19.558376

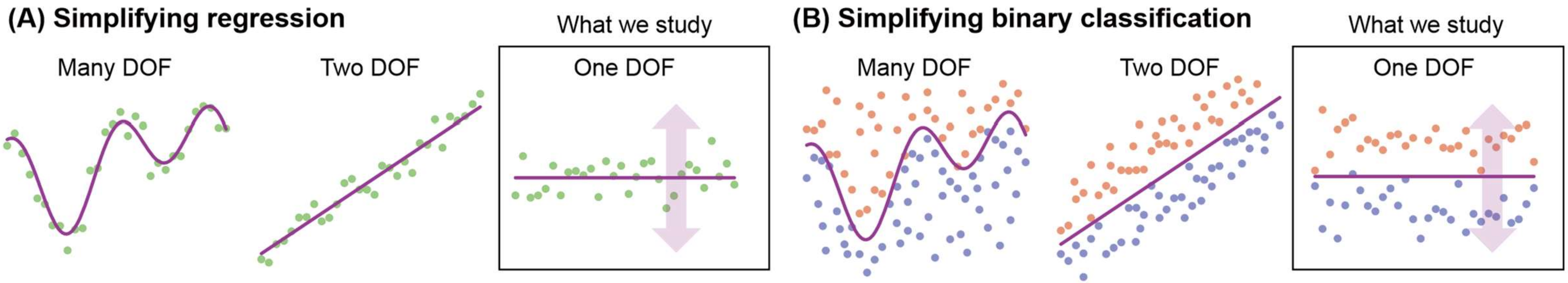
One way to minimize the influence of outliers is to use a cost function that minimizes the influence of distant points.
Ordinary least squares, as its name shows, minimizes the sum of the **squares** of the distance of the model to the data (L2 norm).
This choice of a “loss function” leads to averaging the data points: the **mean** is the optimal solution.
Robust regression often relies on another loss function, the sum of the **absolute distance** of the model to the data (L1 norm).
This choice leads to a more robust solution, less influenced by extreme datapoints: the **median.**

Intermediate choices are possible : in general, the loss function could be the Lp norm with any exponent p :
(or yet other losses, such as cross-entropy).

$$\sum_{i=1}^{N} |\boldsymbol{y}(i) - \hat{y}|^{p}$$

Here the authors attempt to estimate the exponent p from human behavioral data in two tasks :

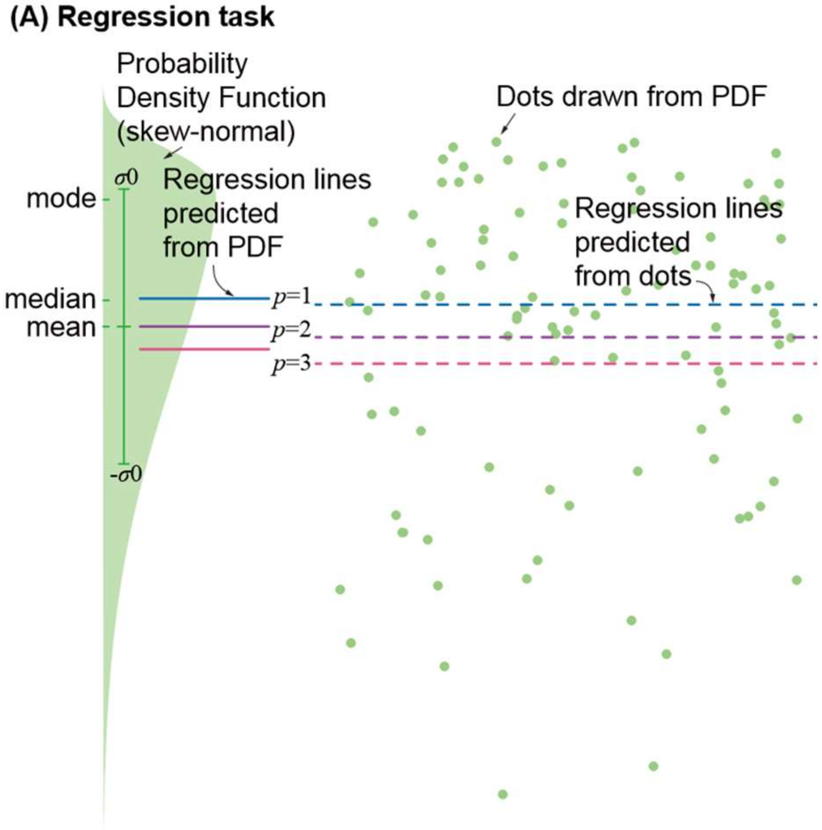
Simplifying regression and binary classification into one degree of freedom (DOF) problems



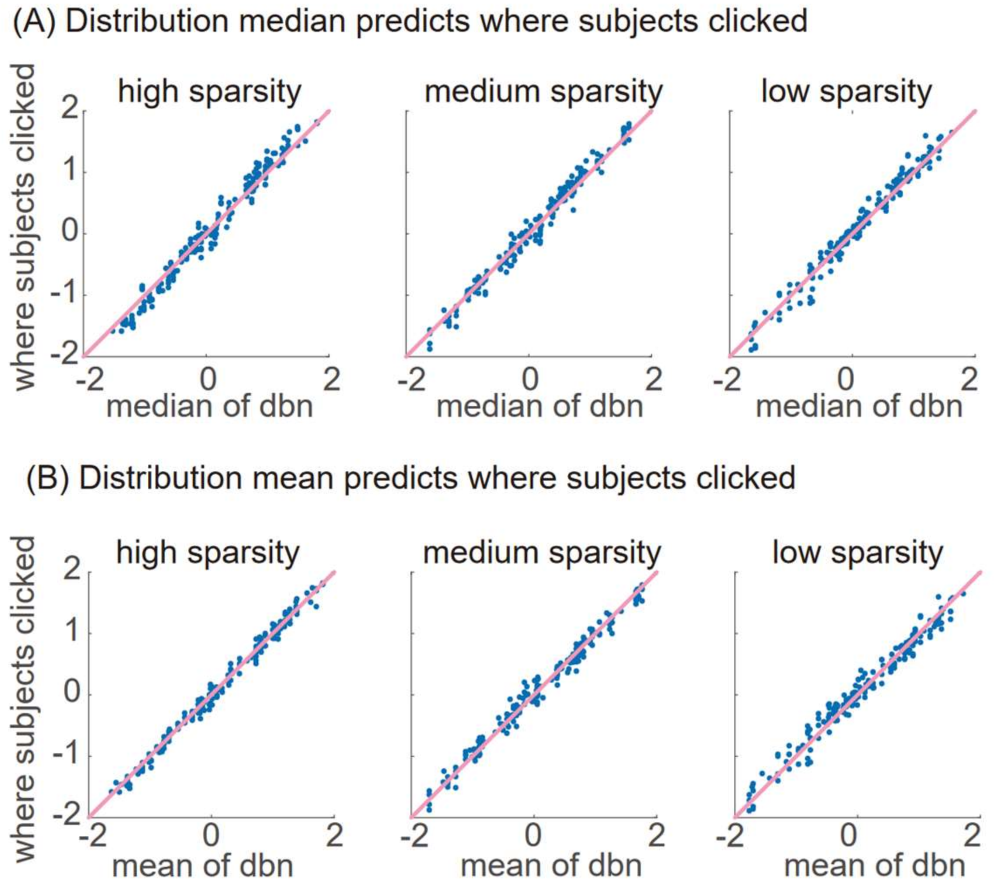
A related question: what is the "loss function" for graphic perception?

Ryu, H. X., & Srinivasan, M. (2023). What Loss Functions Do Humans Optimize When They Perform Regression and Classification. bioRxiv: The Preprint Server for Biology, 2023.09.19.558376. https://doi.org/10.1101/2023.09.19.558376

The data are generated from a « skewed Gaussian » so that the predictions differ as a function of parameter p.
The number of dots is also varied (20, 400 or 8000),



First result: humans are excellent in estimating a central tendency – and their results correlate tightly with both mean and median:

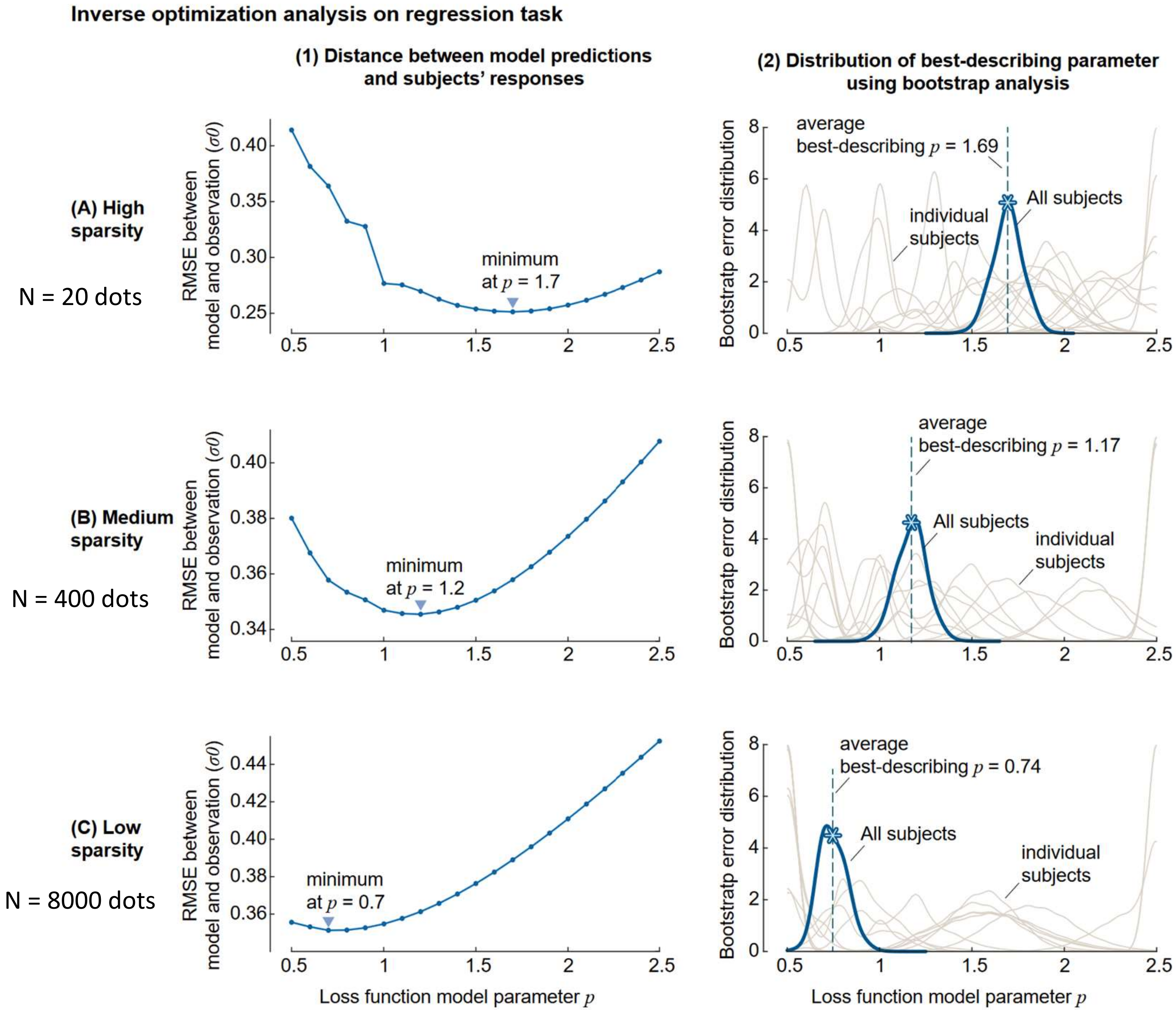


Second result: Humans use an Lp norm which is more robust than ordinary least squares – but whose exponent varies with the number of data points !

A possible interpretation is that, for larger the number of dots, subjects are better able to determine that the distribution is skewed, and therefore the less they rely on the mean as an estimate.

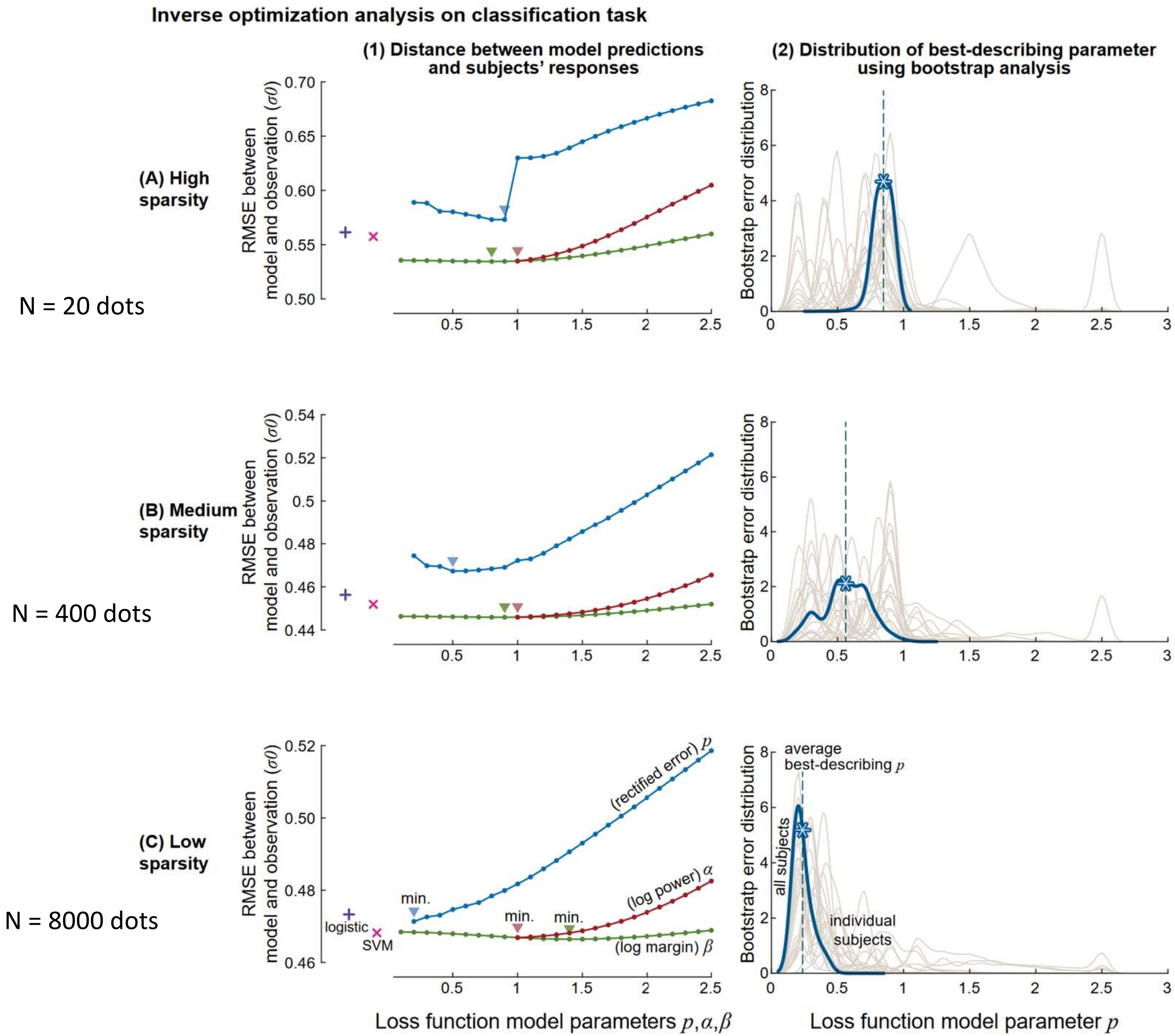
That makes a lot of sense: it is harder to decide that a dot is an outlier when the distribution isn't well sampled and therefore unknown.

Interestingly, and compatible with this idea, decisions were fast (~4 seconds) and slightly *faster* with more dots – showing clearly that subjects were using a parallel, intuitive visual process.



For classification, the results are similar, but more complex:

- Assuming that the participants use an Lp norm for the distance of the *misclassified* data points to the boundary, then the exponent again varies with the number of data points, in a somewhat similar manner
- However, this “rectified error” model isn’t the best one – obviously, the participants also take into account the distance of the *correctly* classified data points.
- Human subjects perform close to a Support Vector Machine (SVM), one of the most common machine learning algorithms !

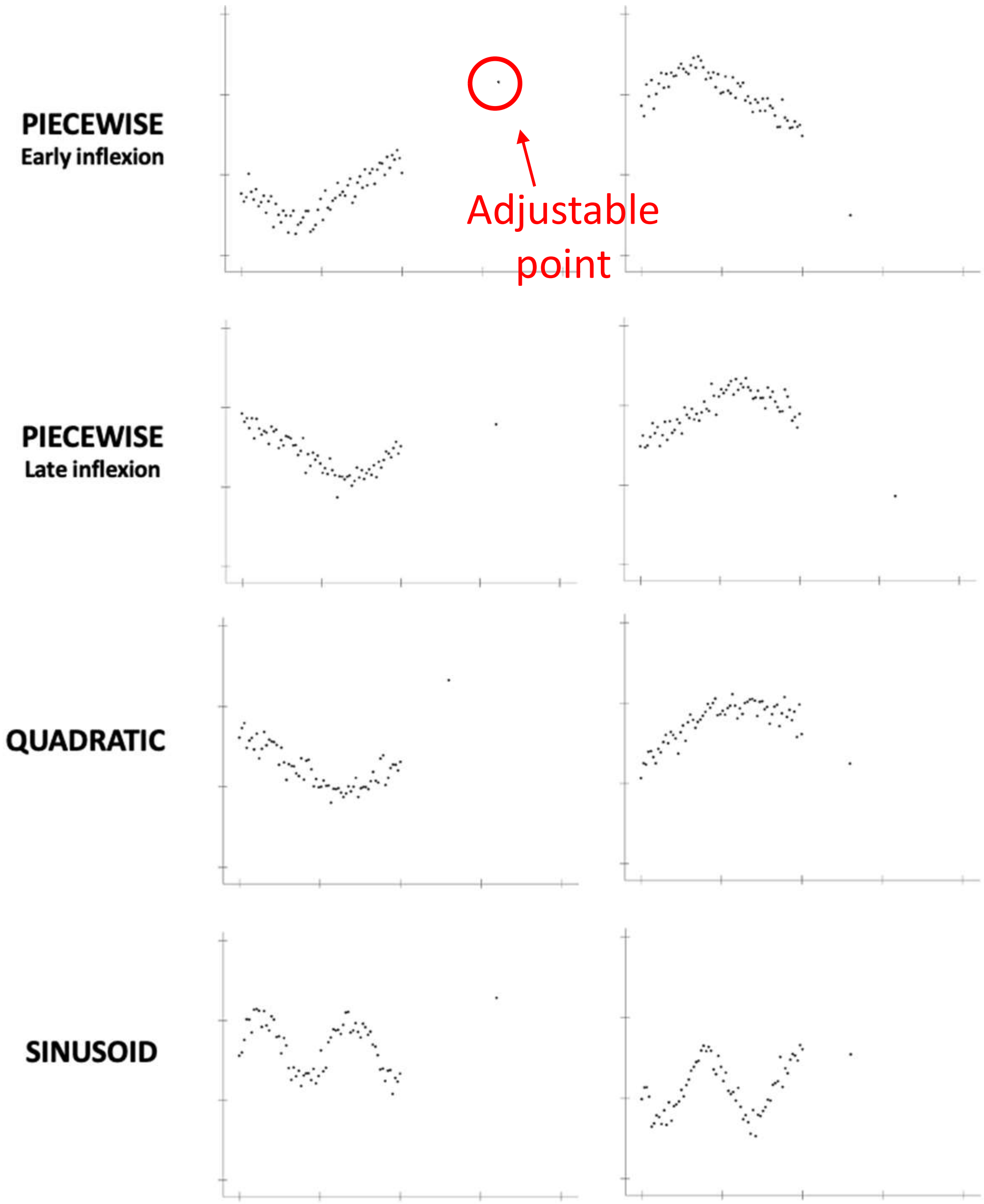


Question 2 : Beyond linear regression, what is the « grammar of functions » that humans can understand?

Ciccione, L., & Dehaene, S. (2021). Can humans perform mental regression on a graph? Accuracy and bias in the perception of scatterplots. *Cognitive Psychology*

We first asked this question first using an extrapolation task.

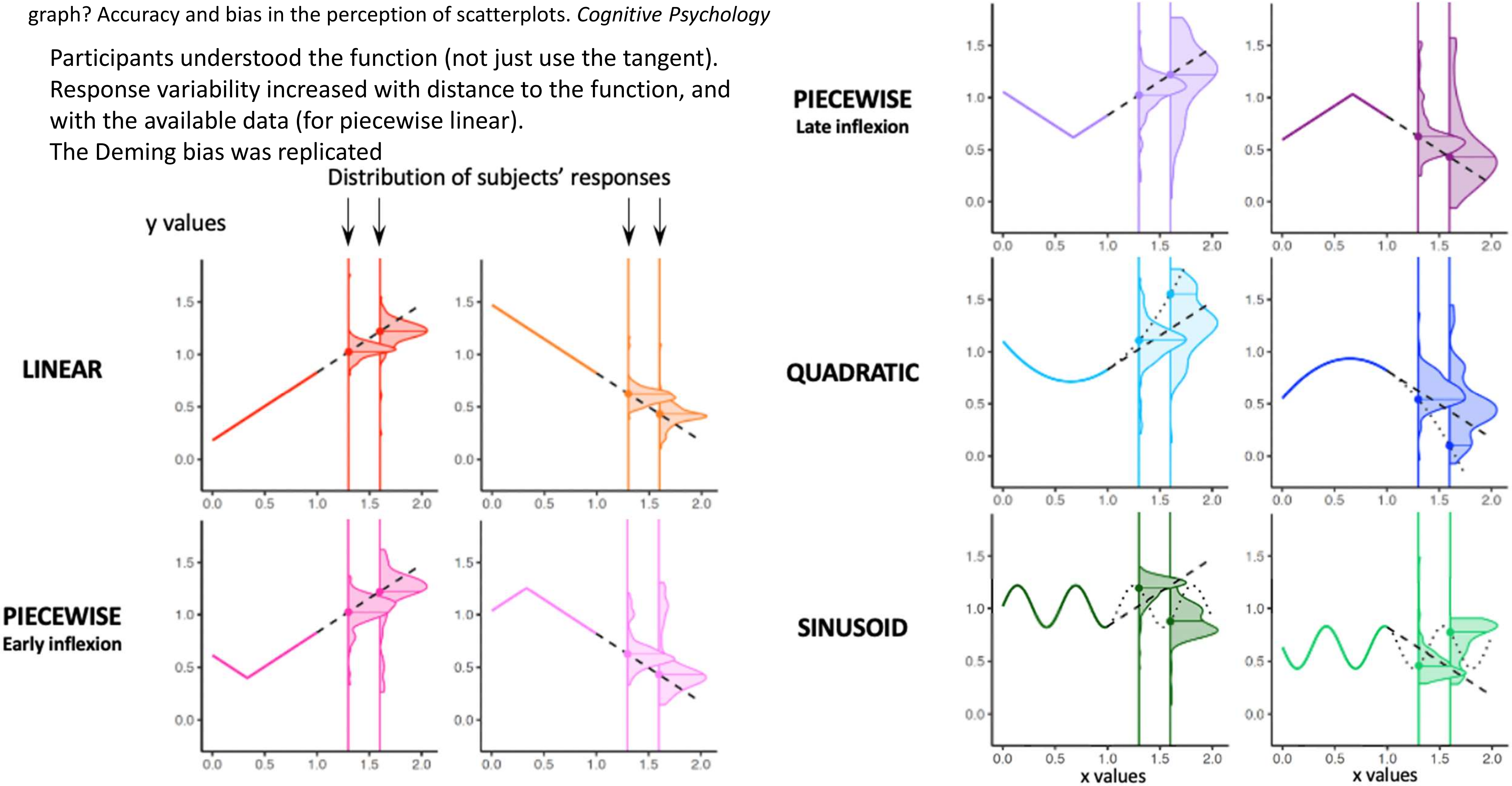
- Subjects are asked to adjust the position of a dot on a vertical axis, in order to place it at the most likely position given the data on the left.
- The left curves can be linear, but also piecewise linear, quadratic, or sinusoid.
- All of these functions have the same derivative at their rightmost point. Thus, if participants adapt their response to the nature of the function, it means that they understand (partially?) the nature of the function and how it should be extrapolated.



Humans can extrapolate a variety of curves

Ciccione, L., & Dehaene, S. (2021). Can humans perform mental regression on a graph? Accuracy and bias in the perception of scatterplots. *Cognitive Psychology*

Participants understood the function (not just use the tangent).
Response variability increased with distance to the function, and with the available data (for piecewise linear).
The Deming bias was replicated



Computational modeling

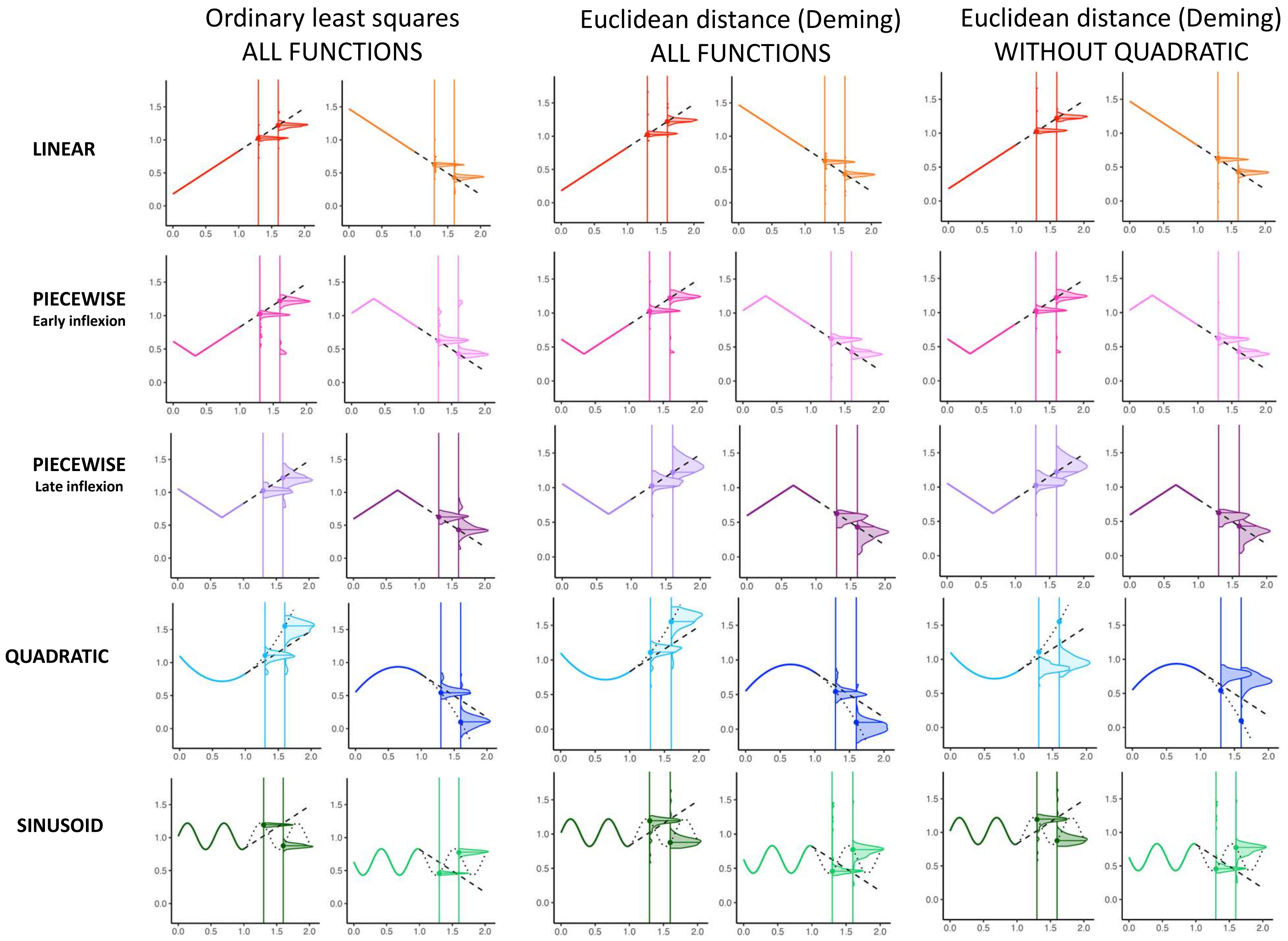
Behavior could be modeled by a **Bayes-optimal observer**.

The model is informed of the **catalog of available functions** and selects the most likely one given the data (with >93% accuracy).

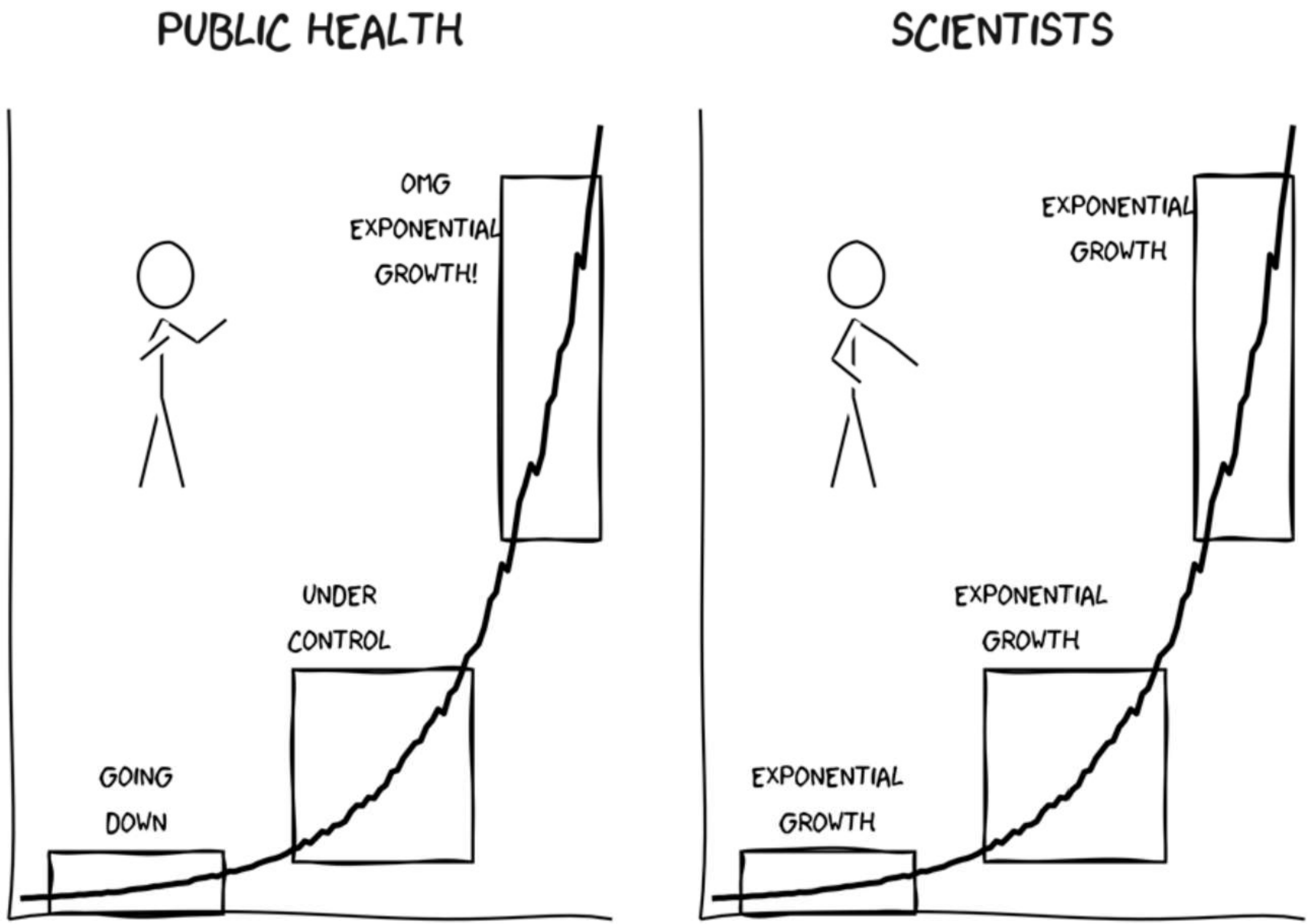
The BIC (Bayesian information criterion) was used, thus penalizing models with a larger number of free parameters.

The Deming distance provided a better fit.

– and this model could partially explain the behavior with quadratics.



And with exponentials? Are people unable to fathom their growth ?



XKCD comics

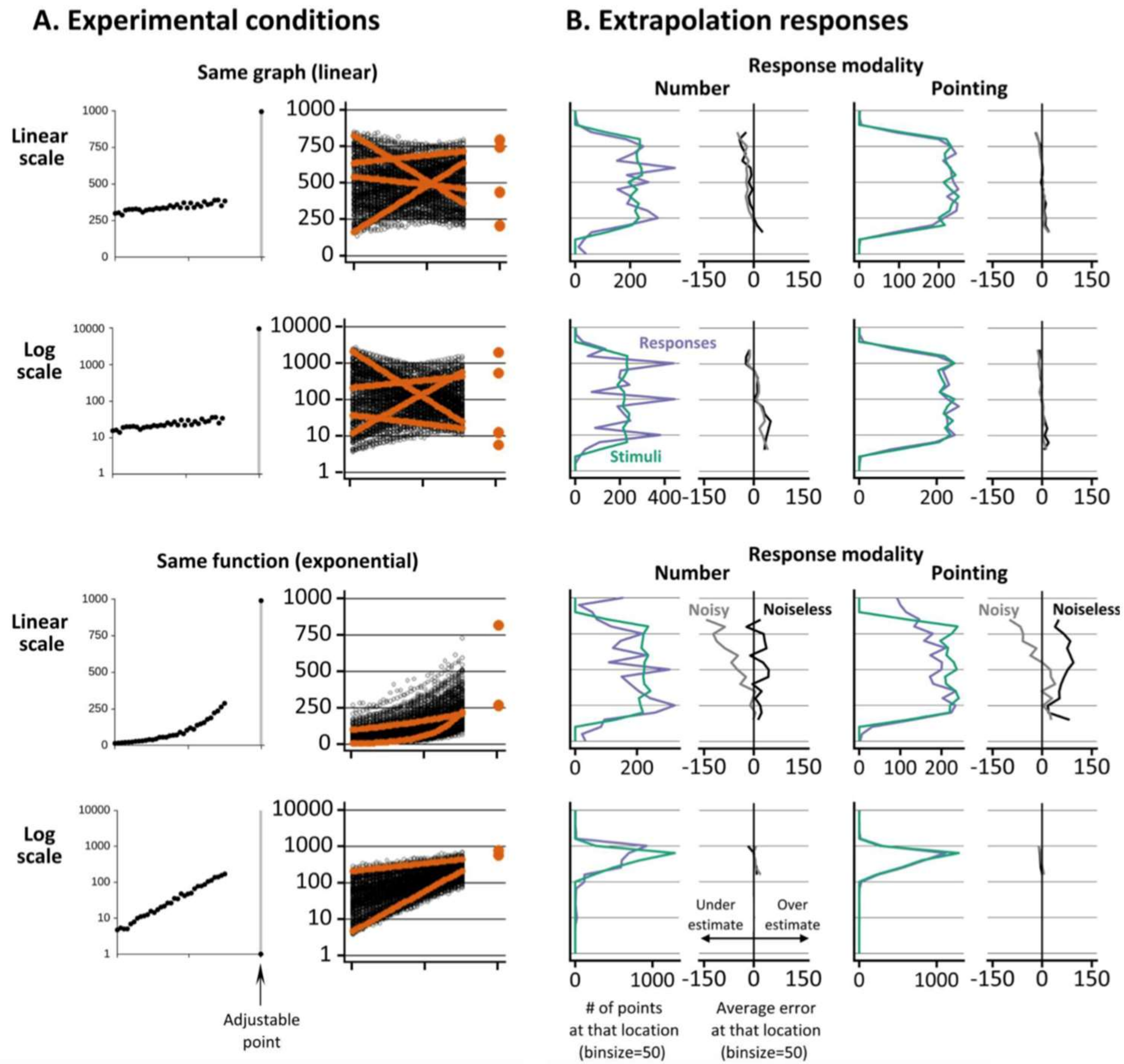
Even with exponentials, adults can be competent when the graph is noiseless

Ciccione, Sablé-Meyer & Dehaene (2022), Analyzing the misperception of exponential growth in graphs, *Cognition*

Questions:
- Are exponentials part of the repertoire of functions of human adults?
- And if perception of exponentials is biased, can we facilitate it?

Four factors were manipulated:
- the function underlying the data (**linear or exponential**)
- the **response modality** (pointing or venturing a number)
- the **scale** on the y axis (linear or logarithmic)
- the amount of **noise** in the data.

Results:
- With linear graphs, responses are precise and relatively unbiased.
- With exponentials, responses **can** be precise when the curve is noiseless.



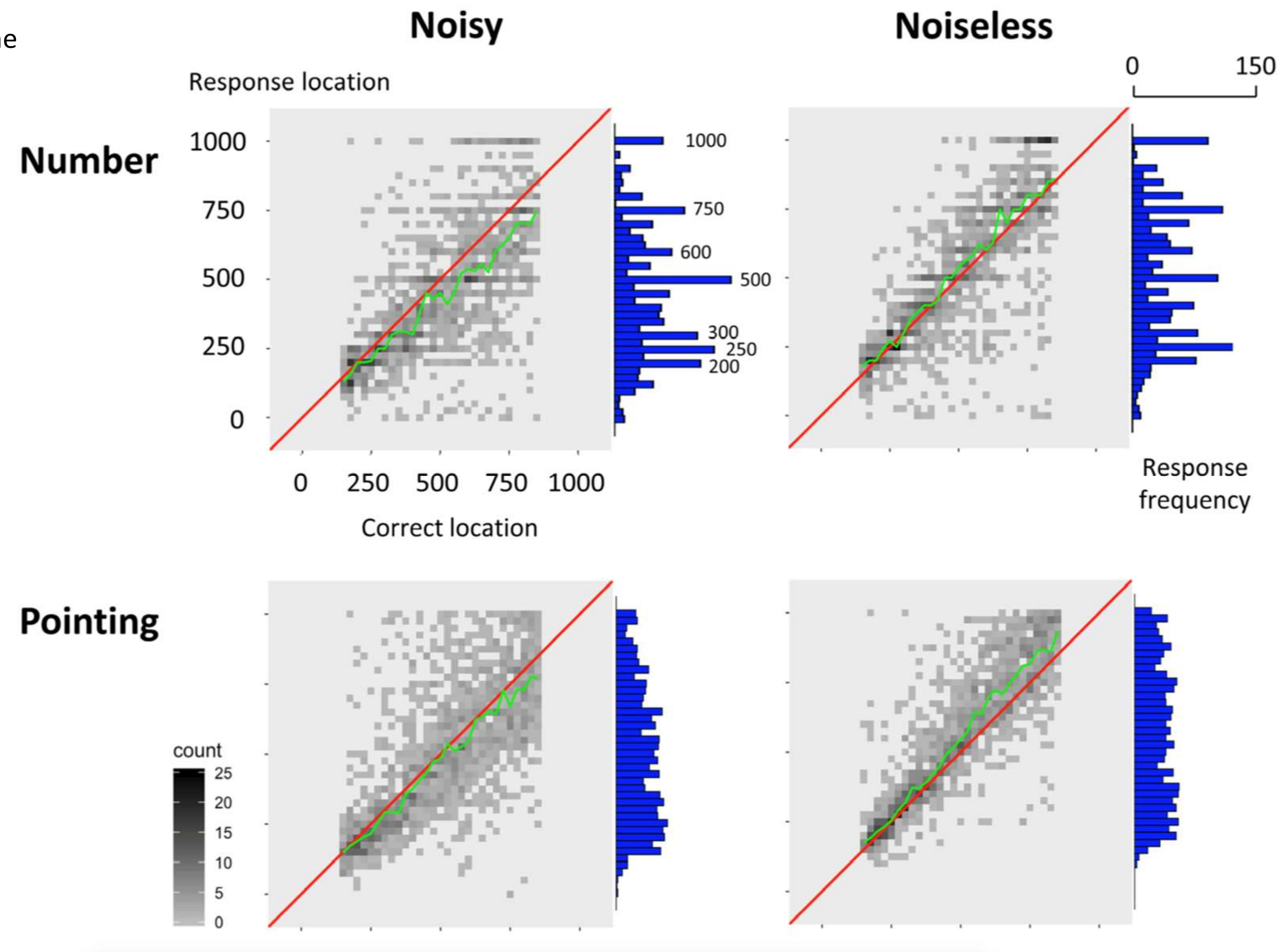
Even with exponentials, adults can be competent when the graph is noiseless

Ciccione, Sablé-Meyer & Dehaene (2022), Analyzing the misperception of exponential growth in graphs, *Cognition*

Responses with numbers are discretized towards round numbers.

For both pointing and numerical responses, the growth of noiseless exponentials is properly perceived.

The misperception of noisy exponentials may be due to a confusion with quadratics, in the presence of noise only.



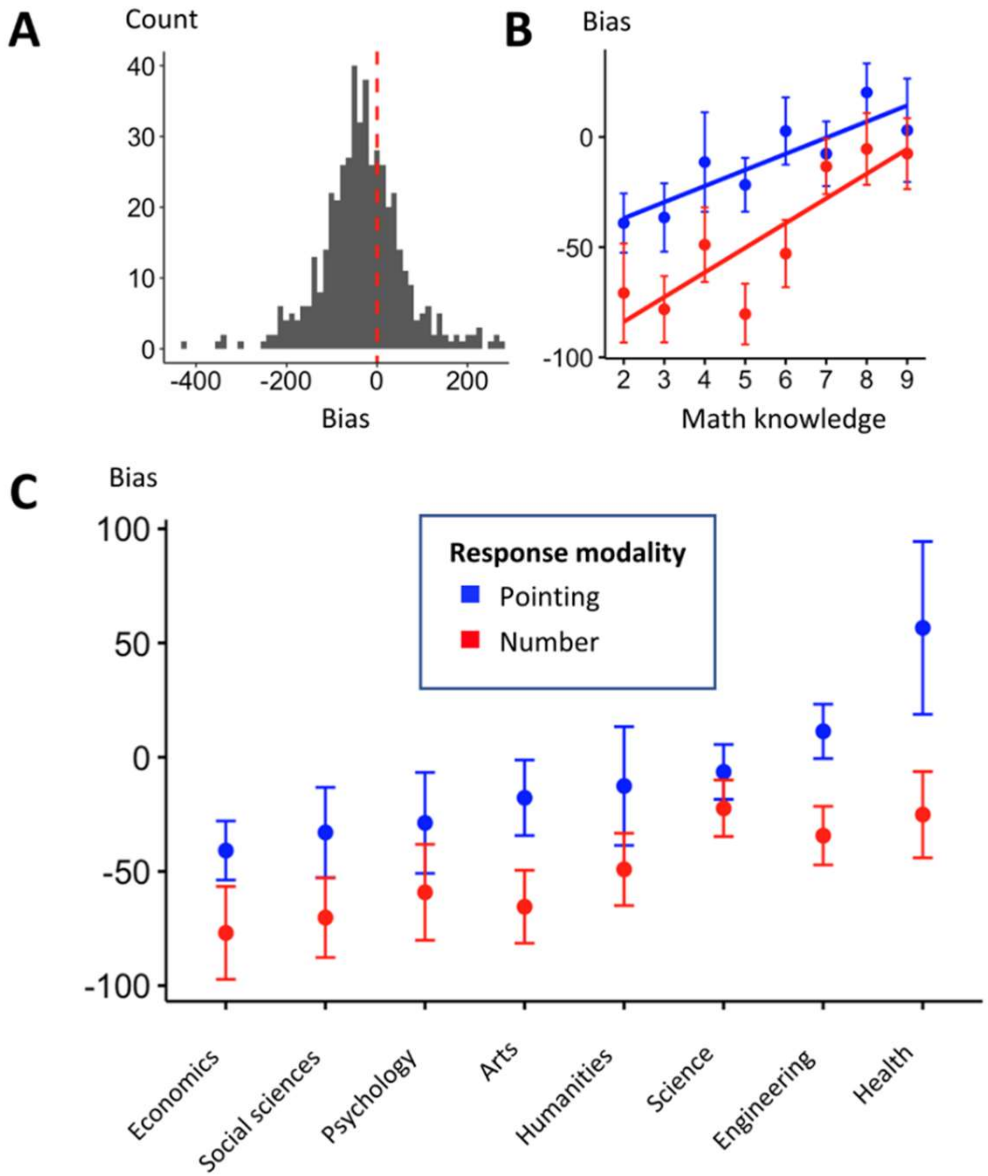
The precision of exponential perception varies with education

Ciccione, Sablé-Meyer & Dehaene (2022), Analyzing the misperception of exponential growth in graphs, *Cognition*

For noisy exponentials, the amount of bias varies with mathematical knowledge and education.

Recommendations :

- Improve education to exponentials
- Reduce the noise in the data plot
    - For instance by plotting the best-fitting exponential on top of the data points
- Use pointing rather than numerical responses
- OR use a log scale with a high density of labels



Summary and conclusion
Grasping the compositional syntax of graphics

The cognitive study of the “grammar of graphics” is in its early stages, but we can already conclude that

- Participants do no just compute naïve statistics such as linear regression over an entire graph
- They can reject outliers
- They understand non-linear curves, such as quadratics, exponentials, sinusoids and bilinear curves.
- Next week we will see that a **language of functions, with compositionality,** is available even to young children.